

CLASS 10 - MATHS

FORMULA BOOK

REAL NUMBERS

- Euclid's Division Lemma
 $a = b \times q + r, 0 \leq r < b.$
- For any two positive integers a and b
 $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$
- For three numbers a, b & c
 (i) $\text{HCF}(a, b, c) \mid \text{LCM}(a, b, c)$ s $a \mid b \mid c$ where a, b, c are positive integers.
 (ii) $\text{LCM}(a, b, c) = \frac{a \mid b \mid c \mid \text{HCF}(a, b, c)}{\text{HCF}(a, b) \mid \text{HCF}(b, c) \mid \text{HCF}(a, c)}$
 (iii) $\text{HCF}(a, b, c) = \frac{a \mid b \mid c \mid \text{LCM}(a, b, c)}{\text{LCM}(a, b) \mid \text{LCM}(b, c) \mid \text{LCM}(a, c)}$

POLYNOMIALS

- Remainder Theorem :** Let $p(x)$ be any polynomial of degree greater than or equal to 1 and a be any real number, if $p(x)$ be divided by linear polynomial $(x - a)$, then the remainder is equal to $p(a)$.
- Factor Theorem :** If $p(x)$ is a polynomial of degree greater than or equal to 1 and a be any real number such that
 (i) if $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$ and
 (ii) if $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$
- Division Algorithm for Polynomial :** $p(x) = q(x) \times g(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) < \text{degree of } q(x)$.
- If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then

$$\alpha + \beta = -\frac{b}{a} \quad \text{and} \quad \alpha\beta = \frac{c}{a}$$

- If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = -\frac{b}{a}, \quad \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} \quad \text{and} \quad \alpha\beta\gamma = -\frac{d}{a}$$

SOME USEFUL IDENTITIES

- (i) $(x + y)^2 = x^2 + y^2 + 2xy$
 (ii) $(x - y)^2 = x^2 + y^2 - 2xy$
 (iii) $(x + y)(x - y) = x^2 - y^2$
 (iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$
 (v) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
 (vi) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
 $= x^3 + 3x^2y + 3xy^2 + y^3$
 (vii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
 $= x^3 - 3x^2y + 3xy^2 - y^3$
 (viii) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
 If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$
 (ix) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
 (x) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

- If a pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ represents :
 (i) Intersecting lines then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (one solution)
 (ii) Parallel lines, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (no solution)

- (iii) Coincident lines, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
(infinitely many solutions)

QUADRATIC EQUATIONS

- Roots of the quadratic equation $ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$ and $a \neq 0$ is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; D = b^2 - 4ac$$

- Nature of Roots**
 - (i) If $D > 0$, distinct and unequal real roots.
 - (ii) If D is a perfect square, the equation has unequal-rational roots.
 - (iii) If $D = 0$, real and equal roots and each root is $-\frac{b}{2a}$.
 - (iv) If $D < 0$, no real roots.
- Formation of a quadratic equation
 $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

ARITHMETIC PROGRESSIONS

- The n^{th} term a_n of an A.P. is
 $a_n = a + (n - 1)d$;
 a = first term
 n = number of terms
 d = common difference
- The sum to n terms of an A.P.

$$S_n = \frac{n}{2} \{2a + (n - 1)d\}$$

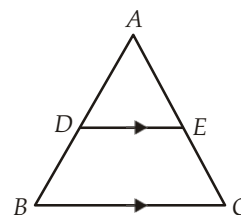
$$\text{Also, } S_n = \frac{n}{2} \{a + l\}$$

TRIANGLES

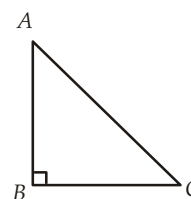
- Basic Proportionality Theorem (B.P.T.) (Thales Theorem)** : In a triangle, a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the same ratio. In $\triangle ABC$, if $DE \parallel BC$.
Then (i) $\frac{AD}{DB} = \frac{AE}{EC}$

$$(ii) \frac{AB}{AD} = \frac{AC}{AE}$$

$$(iii) \frac{AB}{DB} = \frac{AC}{EC}$$



- AAA Similarity Criterion** : If two triangles are equiangular, then they are similar.
- AA Similarity Criterion** : If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
- SSS Similarity Criterion** : If the corresponding sides of two triangles are proportional, then they are similar.
- SAS Similarity Criterion** : If in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.
- Area of Similar Triangles** : The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides, altitudes, medians, angle bisector segments.
- The Pythagoras Theorem** :
In a right triangle, the square of the hypotenuse is equal to the sum of the square of other two sides.
In the given figure,
 $AC^2 = AB^2 + BC^2$.



CO-ORDINATE GEOMETRY

- If $x \leq y$, then $(x, y) \leq (y, x)$.
- If $(x, y) = (y, x)$, then $x = y$.
- Distance between the points $A(x_1, y_1)$, $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- If A, B and C are collinear, then $AB + BC = AC$ or $AC + CB = AB$ or $BA + AC = BC$.
- The points which divides the line segment joining the points $A(x_1, y_1)$, $B(x_2, y_2)$ in the ratio $l : m$

(i) Internally : $\left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m} \right); (l+m \neq 0)$

(ii) Externally: $\left(\frac{lx_2 - mx_1}{l-m}, \frac{ly_2 - my_1}{l-m} \right); (l \neq m)$

- The mid-point of the line segment joining

$$A(x_1, y_1), B(x_2, y_2) \text{ is } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

- Centroid of a ΔABC , with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right).$$

- The area of the triangle formed by the points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$

$$= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AC}{AB}$$

$$\cot \theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{AC}$$

$$\sec \theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{BC}{AB}$$

$$\operatorname{cosec} \theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{BC}{AC}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} \text{ or } \sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \text{ or } \cos \theta = \frac{1}{\sec \theta}$$

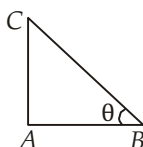
$$\cot \theta = \frac{1}{\tan \theta} \text{ or } \tan \theta = \frac{1}{\cot \theta}$$

INTRODUCTION TO TRIGONOMETRY

- Trigonometric Ratios in ΔABC

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AC}{BC}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{BC}$$



$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

θ T-ratios	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

- Trigonometric Ratios of Complementary Angles**

$$\sin(90^\circ - \theta) = \cos \theta, \quad \cos(90^\circ - \theta) = \sin \theta$$

$$\tan(90^\circ - \theta) = \cot \theta, \quad \cot(90^\circ - \theta) = \tan \theta$$

$$\sec(90^\circ - \theta) = \operatorname{cosec} \theta, \quad \operatorname{cosec}(90^\circ - \theta) = \sec \theta$$

- Trigonometric Identities**

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

CIRCLES

- Tangent to a circle at a point is perpendicular to the radius through the point of contact.
- From a point, lying outside a circle, two and only two tangents can be drawn to it.
- The lengths of two tangents drawn from an external point are equal.

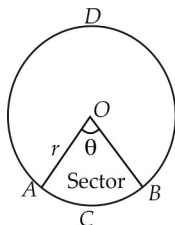
AREAS RELATED TO CIRCLE

- Circumference of a circle = $2\pi r$, where r is the radius of the circle.
- Perimeter of a semicircle with radius r is $2r + \pi r$.
- Area of a circle with radius r is given by $A = \pi r^2$.
- Area of a semicircle of radius $r = \frac{\pi r^2}{2}$.
- Area of a ring whose outer and inner radii are R and r respectively
 $= \pi(R^2 - r^2) = \pi(R + r)(R - r)$
- Perimeter of sector $OACBO = 2r + \frac{2\pi r \theta}{360^\circ}$.
- Area of minor sector $OACBO = \frac{\pi r^2 \theta}{360^\circ}$.

Also, the area of a sector is given by $A = \frac{1}{2}lr$,

where $l = \frac{\pi r \theta}{180^\circ} = \text{length of arc } ACB$.

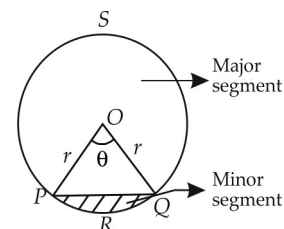
- Area of major sector $OADBO = \pi r^2 - \text{area of minor sector } OACBO$.



- Area of the minor segment $PRQP$

$$= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin \theta$$

- Area of major segment $PSQP$
 $= \pi r^2 - \text{area of minor segment } PRQP$.



SURFACE AREAS AND VOLUMES

- Cube**

If a be the edge of a cube, then

$$\text{Volume} = a^3$$

$$\text{Total surface area} = 6a^2$$

$$\text{Area of four walls} = 4a^2$$

$$\text{Diagonal of cube} = \sqrt{3} \times \text{Edge} = \sqrt{3} a$$

$$\text{Edge of a cube} = \sqrt[3]{\text{Volume}}$$

- Cuboid**

If l be the length, b be the breadth and h be the height of the cuboid, then

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height} = l \times b \times h$$

$$\text{Total surface area} = 2(lb + bh + hl)$$

$$\text{Area of four walls of a room} = 2 \times (l + b)h$$

$$\text{Diagonal of a cuboid} = \sqrt{l^2 + b^2 + h^2}$$

- Cylinder**

If r be the radius of the cylinder and h be the height of the cylinder, then

$$\text{Volume} = \pi r^2 h$$

$$\text{Curved surface area} = 2\pi r h$$

$$\text{Total surface area} = 2\pi r(r + h)$$

- Hollow Cylinder**

If R is the outer radius, r is the inner radius and h be the height of the hollow cylinder, then

$$\text{Volume} = \pi(R^2 - r^2)h$$

$$\text{Total surface area} = 2\pi(R + r)(h + R - r)$$

- Cone**

If r , h and l denote respectively the radius of base, height and slant height of a right circular cone, then

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$\text{Curved surface area} = \pi r l = \pi r \left(\sqrt{h^2 + r^2} \right)$$

Total surface area = curved surface area + area of the base = $\pi r l + \pi r^2 = \pi r(l + r)$

- Sphere**

If r is the radius of the sphere, then

$$\text{Surface area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3} \pi r^3$$

- Hollow Sphere**

If R is the outer radius and r is the inner radius of the hollow sphere, then

$$\text{Volume} = \frac{4}{3} \pi (R^3 - r^3)$$

- Hemisphere**

If r is the radius of the hemisphere, then

$$\text{Curved surface area} = 2\pi r^2$$

$$\text{Total surface area} = 3\pi r^2$$

$$\text{Volume} = \frac{2}{3} \pi r^3$$

- Frustum of a Cone**

If h is the height, l the slant height and r_1 and r_2 the radii of the circular bases ($r_1 > r_2$) of a frustum of a cone, then

$$\text{Volume} = \frac{\pi}{3} (r_1^2 + r_1 r_2 + r_2^2) h$$

$$\text{Lateral surface area} = \pi (r_1 + r_2) l$$

$$\text{Total surface area} = \pi \left\{ (r_1 + r_2) l + r_1^2 + r_2^2 \right\}$$

$$\text{Slant height of the frustum, } l = \sqrt{h^2 + (r_1 - r_2)^2}$$

STATISTICS

- Range** : Highest observation – Lowest observation
- Class size** : Upper class limit – Lower class limit

- Class marks :**

$$\frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

- For Ungrouped Data**

- (i) **Mean**

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

- (ii) **Median**

Case-I : If the number of items n in the data is odd, then

$$\text{Median} = \text{value of } \left(\frac{n+1}{2} \right)^{\text{th}} \text{ item.}$$

Case-II : If the total number of items n in the data is even, then

$$\text{Median} = \frac{1}{2} \times \text{value of } \left[\left(\frac{n}{2} \right)^{\text{th}} + \left(\frac{n}{2} + 1 \right)^{\text{th}} \right] \text{ item}$$

- (iii) **Mode** = 3 Median – 2 Mean

- For Grouped Data**

- (i) **Mean (Direct Method)**

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

- (ii) **Mean (Mean Deviation Method)**

$$\bar{x} = a + \frac{\sum f_i (x_i - a)}{\sum f_i} = a + \frac{\sum f_i d_i}{\sum f_i},$$

where, a = assumed mean,

$\sum f_i$ = total frequency, $d_i = x_i - a$.

- (iii) **Mean (Step Deviation Method)**

$$\bar{x} = a + \frac{\sum f_i \left(\frac{x_i - a}{h} \right)}{\sum f_i} \times h = a + h \left(\frac{\sum f_i u_i}{\sum f_i} \right)$$

where, a = assumed mean, $\sum f_i$ = total frequency,

$$u_i = \frac{x_i - a}{h} = \text{class-size, } u$$

$$(iv) \text{ Median}(M_e) = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h,$$

where, l = lower limit of the median class,
 n = number of observations,

cf = cumulative frequency of the class preceding the median class,

f = frequency of the median class,

h = class size.

•
$$\text{Mode}(M_o) = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h,$$

where, l = lower limit of modal class,

h = size of the class-interval,

f_1 = frequency of the modal class,

f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class succeeding the modal class.

PROBABILITY

- Probability of an event $E = P(E) = \frac{n(E)}{n(S)}$
- $P(E) + P(\bar{E}) = 1$
- For an event E , we have $0 \leq P(E) \leq 1$.