

Exercise 6.1

1. Fill in the blanks using the correct word given in brackets:

- (i) All circles are _____. (congruent, similar)
- (ii) All squares are _____. (similar, congruent)
- (iii) All _____ triangles are similar. (isosceles, equilateral)
- (iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _____ and (b) their corresponding sides are _____. (equal, proportional)

Sol. (i) similar

(ii) similar

(iii) equilateral

(iv) equal, proportional.

2. Give two different examples of pair of

(i) similar figures

(ii) non-similar figures.

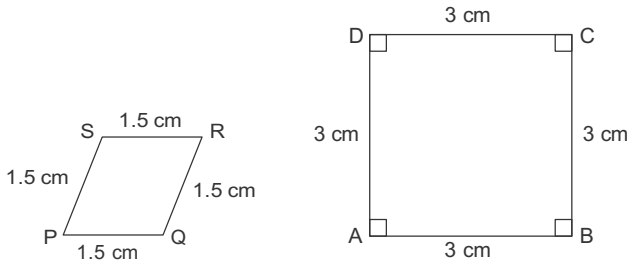
Sol. (i) (a) A passport size photograph and a postcard size photograph of the same person from the same negative.

(b) Equilateral triangles.

(ii) (a) A square and a rhombus.

(b) A circular dinning table and a rectangular dinning table.

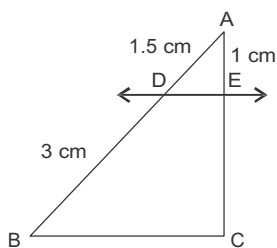
3. State whether the following quadrilaterals are similar or not:



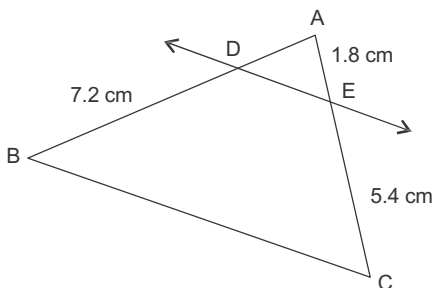
Sol. Not similar as corresponding angles are not equal.

Exercise 6.2

1. In figure, (i) and (ii), $DE \parallel BC$. Find EC in (i) and AD in (ii).



(i)



(ii)

Sol. In figure (i), $\frac{AD}{BD} = \frac{AE}{EC} \Rightarrow \frac{1.5}{3} = \frac{1}{EC} \Rightarrow EC = 2 \text{ cm}.$

In figure (ii), $\frac{AD}{BD} = \frac{AE}{EC} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4} \Rightarrow AD = 2.4 \text{ cm}.$

2. *E and F are points on the sides PQ and PR respectively of a ΔPQR . For each of the following cases, state whether $EF \parallel QR$:*

(i) $PE = 3.9 \text{ cm}, EQ = 3 \text{ cm}, PF = 3.6 \text{ cm}$ and $FR = 2.4 \text{ cm}$

(ii) $PE = 4 \text{ cm}, QE = 4.5 \text{ cm}, PF = 8 \text{ cm}$ and $RF = 9 \text{ cm}$

(iii) $PQ = 1.28 \text{ cm}, PR = 2.56 \text{ cm}, PE = 0.18 \text{ cm}$ and $PF = 0.36 \text{ cm}.$

Sol. (i) $\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$

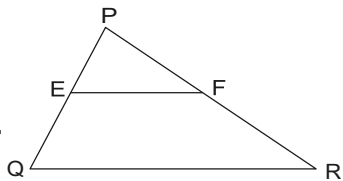
$$\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = 1.5$$

Here, $\frac{PE}{EQ} \neq \frac{PF}{FR}$, so EF is not parallel to QR .

(ii) $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$ and $\frac{PF}{RF} = \frac{8}{9}$

Here, $\frac{PE}{EQ} = \frac{PF}{RF},$

so by inverse of Basic Proportionality Theorem, EF is parallel to QR .



(iii) $\frac{PQ}{PE} = \frac{1.28}{0.18} = \frac{64}{9}$ and $\frac{PR}{PF} = \frac{2.56}{0.36} = \frac{64}{9}$

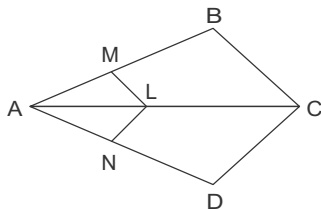
As $\frac{PQ}{PE} = \frac{PR}{PF}$, so EF is parallel to QR .

3. In figure, if $LM \parallel CB$ and $LN \parallel CD$,

prove that $\frac{AM}{AB} = \frac{AN}{AD}.$

Sol. Given: In the given figure, $LM \parallel BC$ and $LN \parallel CD$.

To prove: $\frac{AM}{AB} = \frac{AN}{AD}.$



Proof: In $\triangle ABC$, $LM \parallel BC$

$$\Rightarrow \frac{AM}{MB} = \frac{AL}{LC} \quad \dots(i) \text{ [Basic Proportionality theorem]}$$

In $\triangle ACD$, $LN \parallel CD$

$$\Rightarrow \frac{AN}{ND} = \frac{AL}{LC} \quad \dots(ii) \text{ [Basic Proportionality theorem]}$$

From (i) and (ii), we get

$$\Rightarrow \frac{AM}{MB} = \frac{AN}{ND} \Rightarrow \frac{MB}{AM} = \frac{ND}{AN}$$

$$\Rightarrow \frac{MB}{AM} + 1 = \frac{ND}{AN} + 1$$

$$\Rightarrow \frac{AB}{AM} = \frac{AD}{AN} \Rightarrow \frac{AM}{AB} = \frac{AN}{AD}. \text{ Hence proved.}$$

4. In figure, $DE \parallel AC$ and $DF \parallel AE$.

Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

Sol. Proof: In $\triangle ABC$, $DE \parallel AC$

$$\Rightarrow \frac{BE}{EC} = \frac{BD}{AD} \quad \dots(i) \text{ [Basic Proportionality theorem]}$$

In $\triangle ABE$, $DF \parallel AE$

$$\Rightarrow \frac{BD}{DA} = \frac{BF}{FE} \quad \dots(ii) \text{ [Reason same as above]}$$

From (i) and (ii), we get $\frac{BE}{EC} = \frac{BF}{FE}$. **Hence proved.**

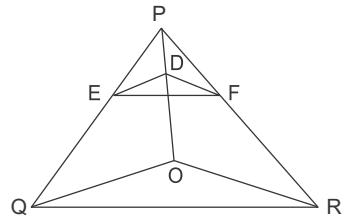
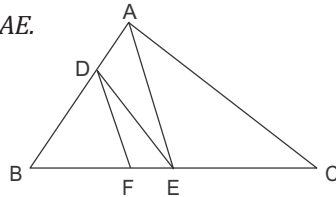
5. In figure, $DE \parallel OQ$ and $DF \parallel OR$.

Show that $EF \parallel QR$.

Sol. Proof: In $\triangle PQO$, $DE \parallel OQ$

$$\therefore \frac{PE}{EQ} = \frac{PD}{DO} \quad \dots(i) \text{ [Basic}$$

Proportionality Theorem]



In $\triangle POR$, $DF \parallel OR$

$$\frac{PD}{DO} = \frac{PF}{FR} \quad \dots(ii) \text{ [Reason same as above]}$$

From (i) and (ii), we get

$$\frac{PE}{EQ} = \frac{PF}{FR} \quad \dots(iii)$$

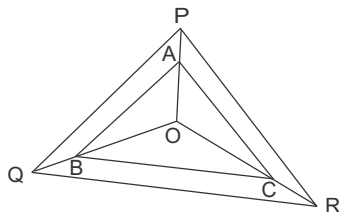
$$\text{In } \triangle PQR, \quad \frac{PE}{EQ} = \frac{PF}{FR} \quad \text{[From (iii)]}$$

$$\Rightarrow EF \parallel QR.$$

[Using converse of Basic Proportionality Theorem]

Hence proved.

6. In figure, A , B and C are points on OP , OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.



Sol. Proof: In $\triangle PQO$, $AB \parallel PQ$

$$\Rightarrow \frac{PA}{AO} = \frac{QB}{BO} \quad \dots(i) \text{ [Basic Proportionality Theorem]}$$

In $\triangle POR$, $AC \parallel PR$

$$\Rightarrow \frac{PA}{AO} = \frac{RC}{CO} \quad \dots(ii) \text{ [Reason same as above]}$$

From (i) and (ii), we have

$$\frac{QB}{BO} = \frac{RC}{CO} \quad \dots(iii)$$

$$\text{In } \triangle QOR, \quad \frac{QB}{BO} = \frac{RC}{CO} \quad \text{[From (iii)]}$$

$$\Rightarrow BC \parallel QR.$$

[Using converse of Basic Proportionality Theorem]

7. Using Basic Proportionality Theorem, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Sol. Given: In a $\triangle ABC$, P is mid-point of AB, i.e., $AP = PB$ and $PQ \parallel BC$.

To prove: Q is mid-point of AC.

Proof: In $\triangle ABC$, $PQ \parallel BC$

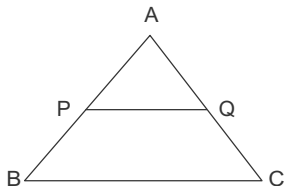
$$\Rightarrow \frac{AP}{PB} = \frac{AQ}{QC} \quad \dots(i) \text{ [Basic Proportionality Theorem]}$$

$$\text{Also, } AP = PB \quad \dots(ii) \text{ [Given]}$$

From (i) and (ii),

$$\Rightarrow 1 = \frac{AQ}{QC}$$

$$\Rightarrow AQ = QC \Rightarrow Q \text{ is mid-point of AC.}$$



8. Using Converse of Basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Sol. Given: In a $\triangle ABC$, P and Q are mid-points of sides AB and AC respectively.

To prove: $PQ \parallel BC$.

Proof: As P and Q are mid-points of AB and AC.

$$\therefore AP = PB \text{ and } AQ = QC \quad \dots(i)$$

$$\Rightarrow \frac{AP}{PB} = \frac{AQ}{QC} \quad \dots(ii) \text{ [From (i)]}$$

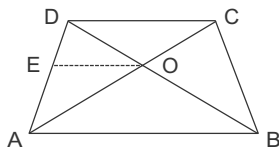
$$\text{In } \triangle ABC, \frac{AP}{PB} = \frac{AQ}{QC} \quad \text{[From (ii)]}$$

$$\Rightarrow PQ \parallel BC.$$

[Using converse of basic Proportionality Theorem]

9. ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the point O. Show

$$\text{that } \frac{AO}{BO} = \frac{CO}{DO}.$$



Sol. Given: A trapezium ABCD, $AB \parallel CD$.

Diagonals AC and BD intersect at O.

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$.

Construction: Draw $OE \parallel AB$, meeting AD at E.

Proof: In $\triangle ADB$, $OE \parallel AB$.

$$\Rightarrow \frac{DE}{EA} = \frac{DO}{BO} \quad \dots(i) \text{ [Basic Proportionality Theorem]}$$

As $AB \parallel OE$ and $AB \parallel DC$

$\therefore OE \parallel DC$

In $\triangle ADC$, $OE \parallel DC$

$$\Rightarrow \frac{DE}{EA} = \frac{CO}{AO} \quad \dots(ii) \text{ [Basic Proportionality Theorem]}$$

From (i) and (ii), we have

$$\frac{CO}{AO} = \frac{DO}{BO} \Rightarrow \frac{AO}{BO} = \frac{CO}{DO} \quad \text{Hence proved.}$$

- 10.** The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Sol. Given: In quadrilateral ABCD, O is the point of intersection AC and BD such that

$$\frac{AO}{BO} = \frac{CO}{DO}$$

To prove: ABCD is a trapezium.

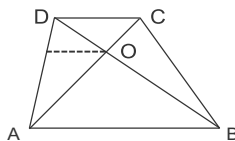
Construction: Draw $OE \parallel AB$

Proof: In $\triangle DAB$, $OE \parallel AB$

$$\frac{OB}{OD} = \frac{AE}{ED} \quad \dots(i) \text{ [Basic Proportionality Theorem]}$$

$$\text{Also } \frac{OA}{OC} = \frac{OB}{OD} \text{ [Given]} \Rightarrow \frac{OA}{OC} = \frac{AE}{ED} \quad \text{[From (i)]}$$

$$\text{In } \triangle ADC, \frac{OA}{OC} = \frac{AE}{ED}$$



$\Rightarrow$ $OE \parallel DC$...(ii)
[Converse of Basic Proportionality Theorem]

Also $OE \parallel AB$(iii) [From construction]

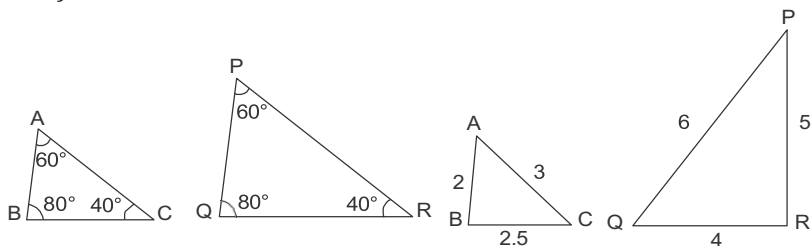
From (ii) and (iii), we have

$DC \parallel AB$

$\therefore$ Quadrilateral ABCD is a trapezium.

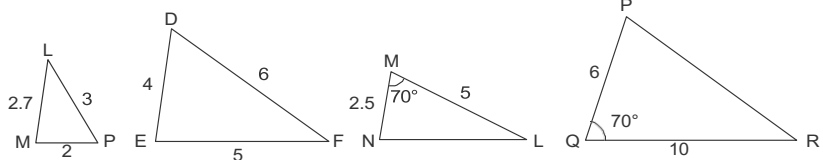
Exercise 6.3

1. State which pairs of triangles in figure are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



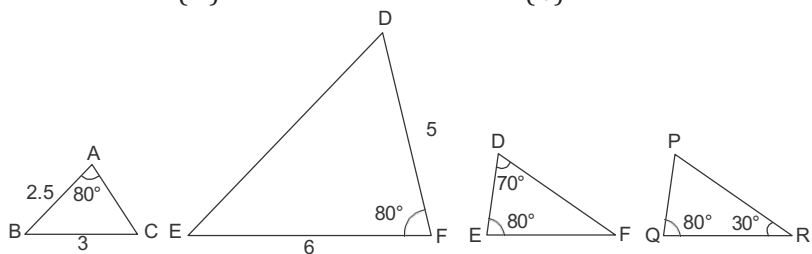
(i)

(ii)



(iii)

(iv)



(v)

(vi)

Sol. (i) $\triangle ABC \sim \triangle PQR$.

[AAA similarity]

$$(ii) \quad \frac{AB}{QR} = \frac{BC}{PR} = \frac{AC}{PQ} = \frac{1}{2}$$

$$\Rightarrow \triangle ABC \sim \triangle QRP. \quad [\text{SSS similarity}]$$

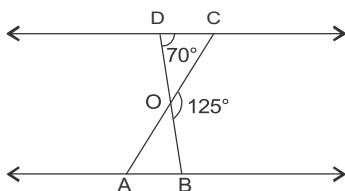
(iii) Not similar, as sides are not in proportional.

(iv) Not similar, SAS not satisfied.

(v) Not similar.

(vi) Similar as $\triangle DEF \sim \triangle PQR$. [AAA similarity]

2. In figure, $\triangle ODC \sim \triangle OBA$,
 $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$
and $\angle OAB$.



Sol. $\triangle ODC \sim \triangle OBA$

$$\Rightarrow \angle OAB = \angle OCD$$

$$\angle DOC = 180^\circ - 125^\circ = 55^\circ$$

$$\angle OCD = 180^\circ - (70^\circ + 55^\circ) = 55^\circ$$

$$\text{And } \angle OAB = 55^\circ$$

[From (i)]

3. Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using a similarity

criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Sol. Consider triangles AOB and COD.

In these triangles,

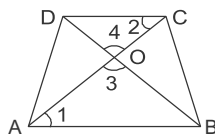
$$\angle 1 = \angle 2 \text{ [Alternate angles]}$$

$$\angle 4 = \angle 3 \text{ [Vertically opposite angles]}$$

$$\therefore \triangle AOB \sim \triangle COD \text{ [AA similarity]}$$

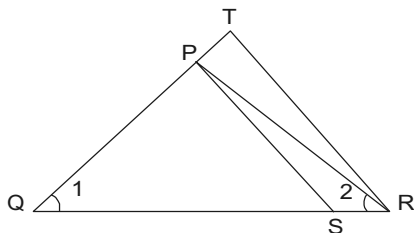
$$\therefore \frac{OA}{OC} = \frac{OB}{OD}$$

Hence proved.



4. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that

$$\triangle PQS \sim \triangle TQR.$$



Sol. In $\triangle PQR$, $\angle 1 = \angle 2 \Rightarrow PQ = PR$... (i)

Given, $\frac{QR}{QS} = \frac{QT}{PR} \Rightarrow \frac{QR}{QS} = \frac{QT}{PQ}$ [Using (i)]

In triangles PQS and QTR,

$$\frac{QR}{QS} = \frac{QT}{PQ} \quad \text{[Proved above]}$$

and $\angle 1$ is common.

$$\therefore \triangle PQS \sim \triangle QTR. \quad \text{[SAS similarity]}$$

5. *S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$. Show that $\triangle RPQ \sim \triangle RTS$.*

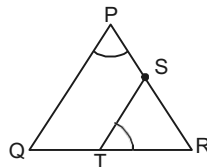
Sol. In triangles PQR and TRS,

$$\angle P = \angle RTS \quad \text{[Given]}$$

and $\angle R$ is common.

$$\therefore \triangle RPQ \sim \triangle RTS$$

[AA similarity]



6. *In figure, if $\triangle ABE \cong \triangle ACD$, show that $\triangle ADE \sim \triangle ABC$.*

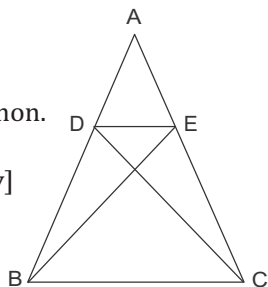
Sol. $\triangle ABE \cong \triangle ACD$

$$\Rightarrow AB = AC \quad \text{and} \quad AE = AD$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} \quad \text{and} \quad \angle A \text{ is common.}$$

$$\Rightarrow \triangle ADE \sim \triangle ABC \quad \text{[SAS similarity]}$$

7. *In figure, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P. Show that:*



(i) $\triangle AEP \sim \triangle CDP$

(ii) $\triangle ABD \sim \triangle CBE$

(iii) $\triangle AEP \sim \triangle ADB$

(iv) $\triangle PDC \sim \triangle BEC$

Sol. (i) In $\triangle AEP$ and $\triangle CDP$,

$$\angle AEP = \angle CDP$$

$$\angle APE = \angle CPD$$

$$\therefore \triangle AEP \sim \triangle CDP$$

(ii) In $\triangle ABD$ and $\triangle CBE$,

$$\angle ADB = \angle CEB$$

and $\angle B$ is common.

$$\therefore \triangle ABD \sim \triangle CBE.$$

(iii) Let $\angle DAB = x$

$$\Rightarrow \angle PAE = x$$

In $\triangle APE$ and $\triangle ADB$,

$$\angle PAE = \angle DAB$$

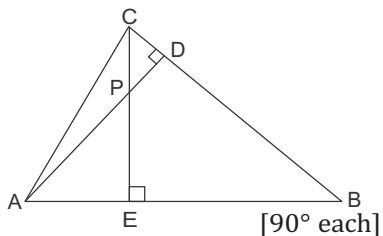
$$\angle AEP = \angle ADB$$

$$\therefore \triangle AEP \sim \triangle ADB$$

(iv) In $\triangle PDC$ and $\triangle BEC$,

$$\angle C \text{ is common and } \angle CDP = \angle CEB$$

$$\therefore \triangle PDC \sim \triangle BEC.$$



[Vertically opposite angles]

[AA similarity]

[90° each]

[AA similarity]

...(i)

...(ii)

[From (i) and (ii)]

[90° each]

[AA similarity]

Hence proved.

8. E is a point on the side AD produced of a parallelogram $ABCD$ and BE intersects CD at F . Show that $\triangle ABE \sim \triangle CFB$.

Sol. Consider $\triangle ABE$ and $\triangle CFB$

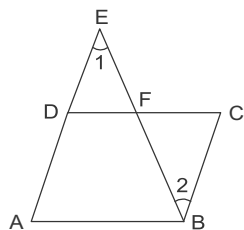
$$\angle A = \angle C$$

[Opposite angles of a parallelogram]

$$\angle 1 = \angle 2 \quad [\text{Alternate angles}]$$

$$\therefore \triangle ABE \sim \triangle CFB \quad [\text{AA similarity}]$$

Hence proved.



9. In figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

$$(i) \triangle ABC \sim \triangle AMP$$

$$(ii) \frac{CA}{PA} = \frac{BC}{MP}$$

Sol. In $\triangle ABC$ and $\triangle AMP$,

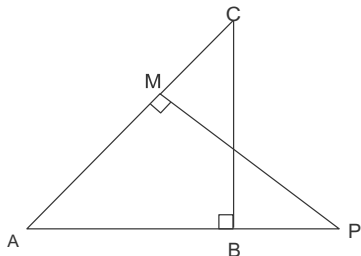
$\angle A$ is common

and $\angle ABC = \angle AMP$.

[90° each]

$$\therefore (i) \triangle ABC \sim \triangle AMP$$

$$(ii) \frac{CA}{PA} = \frac{BC}{MP}$$



[AA similarity]

[Corresponding sides]

Hence proved.

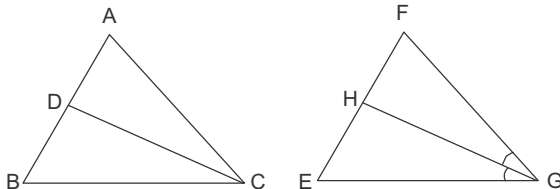
10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle EFG$ respectively. If $\triangle ABC \sim \triangle FEG$, show that:

$$(i) \frac{CD}{GH} = \frac{AC}{FG}$$

$$(ii) \triangle DCB \sim \triangle HGE$$

$$(iii) \triangle DCA \sim \triangle HGF$$

Sol. (i)



$$\triangle ABC \sim \triangle FEG$$

$$\therefore \angle A = \angle F \quad \dots(i)$$

$$\text{and } \angle ACB = \angle FGE$$

$$\Rightarrow \frac{1}{2} \angle ACB = \frac{1}{2} \angle FGE$$

$$\Rightarrow \angle ACD = \angle FGH \quad \dots(ii)$$

[$\because$ CD and GH are the bisectors of $\angle ACB$ and $\angle FGE$ respectively]

$$\therefore \triangle ACD \sim \triangle FGH \quad [\text{AA similarity}] \quad [\text{Using (i), (ii)}]$$

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

(ii) Consider $\triangle DCB$ and $\triangle HGE$

$$\angle B = \angle E \quad \dots(iii) [\because \triangle ABC \sim \triangle FEG]$$

$$\angle ACB = \angle FGE \quad [\because \triangle ABC \sim \triangle FEG]$$

$$\frac{1}{2} \angle ACB = \frac{1}{2} \angle FGE$$

$$\Rightarrow \angle DCB = \angle HGE \quad \dots(iv) [\because CD \text{ and } GH \text{ are the bisectors of } \angle ACB \text{ and } \angle FGE \text{ respectively}]$$

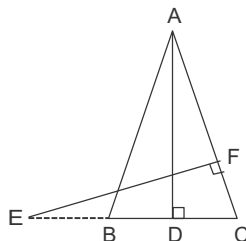
$$\therefore \triangle DCB \sim \triangle HGE$$

...[AA similarity] [From (iii) and (iv)]

(iii) Refer result (i). $\triangle ACD \sim \triangle FGH$

$$\Rightarrow \triangle DCA \sim \triangle HGF.$$

11. In figure, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.



Sol. Consider $\triangle ABD$ and $\triangle ECF$,

$$\angle ADB = \angle EFC$$

[90° each]

$$\angle ABD = \angle C$$

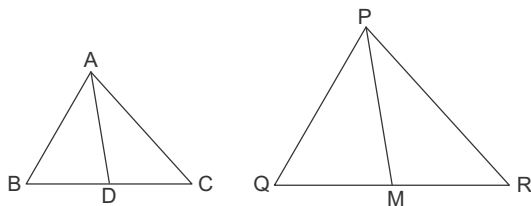
[$\because AB = AC$]

$$\therefore \triangle ABD \sim \triangle ECF.$$

[AA similarity]

Hence proved.

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see figure). Show that $\triangle ABC \sim \triangle PQR$.



Sol.
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

...(i) [Given]

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2} BC}{\frac{1}{2} QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PB}$$

$$\dots(ii) \left(\because BD = \frac{1}{2} BC \text{ and } QM = \frac{1}{2} QR \right)$$

$$\therefore \triangle ABD \sim \triangle PQM$$

[From (ii) SSS similarity]

$$\Rightarrow \angle B = \angle Q$$

...(iii)

In $\triangle ABC$ and $\triangle PQR$,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

[From (i)]

$$\text{and } \angle B = \angle Q$$

[From (iii)]

$$\Rightarrow \triangle ABC \sim \triangle PQR.$$

[SAS similarity]

Hence proved.

13. *D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.*

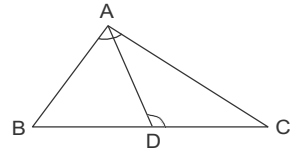
Sol. Consider $\triangle ABC$ and $\triangle ADC$

$$\angle BAC = \angle ADC$$

[Given]

$$\angle C \text{ is common.}$$

$$\therefore \triangle ABC \sim \triangle DAC \text{ [AA similarity]}$$



$$\therefore \frac{CA}{CD} = \frac{CB}{CA} \Rightarrow CA^2 = CB \cdot CD.$$

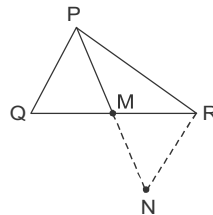
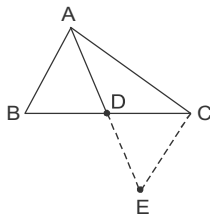
Hence proved.

14. *Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$.*

Sol. Given:
$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}.$$

To prove: $\triangle ABC \sim \triangle PQR$.

Construction: Produce AD and PM to E and N respectively such that $AD = DE$ and $PM = MN$. Join EC and RN.



Proof: Consider $\triangle ABD$ and $\triangle CDE$

$$AD = DE \quad \text{[Construction]}$$

$$BD = DC \quad \text{[D is mid-point of BC]}$$

$$\text{and } \angle ADB = \angle CDE \quad \text{[Vertically opposite angles]}$$

$$\therefore \triangle ABD \cong \triangle CED \quad \text{[SAS]}$$

$$\therefore AB = CE \quad \dots(i)$$

$$\text{Similarly, we can show that } PM = RN \quad \dots(ii)$$

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \text{[Given]}$$

$$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM} \quad \text{[From (i) and (ii)]}$$

$$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN} \quad \text{[Using construction]}$$

$$\Rightarrow \triangle AEC \sim \triangle PNR \quad \text{[SSS similarity]}$$

$$\therefore \angle DAC = \angle MPR \quad \dots(iii)$$

Similarly, we can show that

$$\angle BAD = \angle QPM \quad \dots(iv)$$

$$\Rightarrow \angle BAD + \angle DAC = \angle QPM + \angle MPR \quad \text{[From (iii), (iv)]}$$

$$\Rightarrow \angle BAC = \angle QPR \quad \dots(v)$$

In $\triangle ABC$ and $\triangle PQR$,

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad \dots(vi) \quad \text{[Given]}$$

$$\text{And } \angle BAC = \angle QPR \quad \text{[From (v)]}$$

$$\Rightarrow \triangle ABC \sim \triangle PQR \quad \text{[SAS similarity]}$$

Hence proved.

- 15.** A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

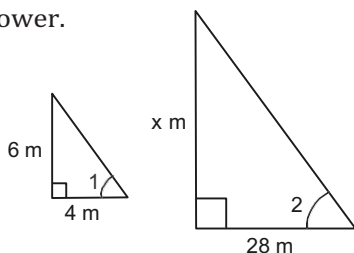
Sol. Let x m be the height of the tower.

Two triangles are similar

as at the same time $\angle 1 = \angle 2$

(see figure)

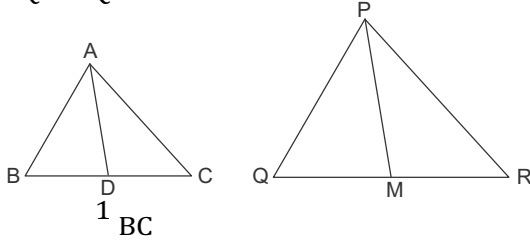
$$\therefore \frac{6}{4} = \frac{x}{28} \Rightarrow x = 42 \text{ m.}$$



16. If AD and PM are medians of triangles ABC and PQR , respectively where $\triangle ABC \sim \triangle PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.
[Given]

Sol. Proof: $\triangle ABC \sim \triangle PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$



$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \quad \dots(i) \quad \left(\because BD = \frac{1}{2}BC \text{ and } QM = \frac{1}{2}QR \right)$$

Also $\angle B = \angle Q$

$\dots(ii) \quad [\because \triangle ABC \sim \triangle PQR]$

From (i) and (ii), we get

$$\triangle ABD \sim \triangle PQM$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$

Hence proved.

Exercise 6.4

1. Let $\triangle ABC \sim \triangle DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2 . If $EF = 15.4 \text{ cm}$, find BC .

Sol. $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{BC^2}{EF^2} \Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$

$$\Rightarrow \frac{8}{11} = \frac{BC}{15.4}$$

$$\Rightarrow BC = 11.2 \text{ cm.}$$

2. Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O . If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD .

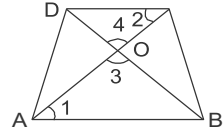
Sol. $\triangle AOB \sim \triangle COD$

...(i)

As $\angle 1 = \angle 2$ [Alternate angles]

and $\angle 3 = \angle 4$ [Vertically opposite angles]

$$\therefore \frac{ar(\triangle AOB)}{ar(\triangle COD)} = \frac{AB^2}{CD^2}$$



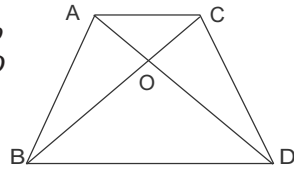
[If triangles are similar, their areas are proportional to the squares of the corresponding sides]

$$= \frac{(2CD)^2}{CD^2} = \frac{4}{1}$$

$$\therefore ar(\triangle AOB) : ar(\triangle COD) = 4 : 1.$$

3. In figure, ABC and DBC are two triangles on the same base BC . If AD intersects BC at O , show that

$$\frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{AO}{DO}.$$



Sol.

$$\frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{\frac{1}{2} BC \cdot AL}{\frac{1}{2} BC \cdot DM}$$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{AL}{DM}$$

But, $\triangle ALO \sim \triangle DMO$

[AA similarity]

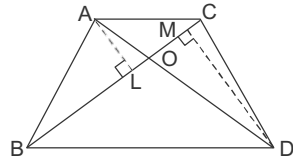
$$\therefore \frac{AL}{DM} = \frac{AO}{DO}$$

...(ii)

From (i) and (ii), we get

$$\frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{AO}{DO}.$$

Hence proved.



...(i)

4. If the areas of two similar triangles are equal, prove that they are congruent.

Sol. Let $\triangle ABC \sim \triangle PQR$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \quad \dots(i)$$

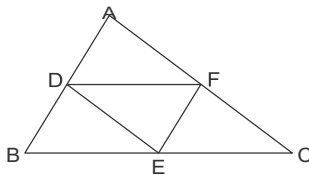
Also $ar(\triangle ABC) = ar(\triangle PQR) \quad \dots(ii) \text{ (given)}$

$$\therefore \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \quad [\text{From (i), (ii)}]$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \Rightarrow \triangle ABC \cong \triangle PQR \quad [\text{SSS similarity}]$$

5. *D, E and F are respectively the mid-points of sides AB, BC and CA of $\triangle ABC$. Find the ratio of the areas of $\triangle DEF$ and $\triangle ABC$.*

Sol.



D and F are the mid-points of sides AB and AC of a $\triangle ABC$.

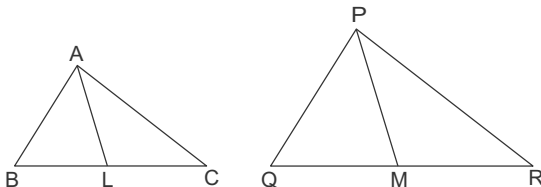
$$\therefore DF \parallel BC \text{ and } DF = \frac{1}{2} BC$$

Also $\triangle EDF \sim \triangle ACB$. [AAA similarity]

$$\begin{aligned} \therefore \frac{ar(\triangle DEF)}{ar(\triangle CAB)} &= \frac{DF^2}{BC^2} = \frac{DF^2}{(2DF)^2} = \frac{1}{4} \\ \therefore ar(\triangle DEF) : ar(\triangle CAB) &= 1 : 4. \end{aligned}$$

6. *Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.*

Sol.



$$\triangle ABC \sim \triangle PQR$$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} \quad \Rightarrow \quad \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BL}{QM} \quad \dots(i) \quad [\because AL \text{ and } PM \text{ are median}]$$

$$\text{Also } \angle B = \angle Q \quad \dots(ii) \quad [\because \triangle ABC \sim \triangle PQR]$$

From (i) and (ii),

$$\triangle ABL \sim \triangle PQM \quad [\text{SAS similarity}]$$

$$\therefore \frac{AB}{PQ} = \frac{AL}{PM}$$

$$\text{Hence } \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AL}{PM} \quad \dots(iii)$$

$$\text{Also } \frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{AB^2}{PQ^2} \quad \dots(iv)$$

[If triangles are similar their areas are proportional to the squares of the corresponding sides.]

From (iii) and (iv), we get

$$\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{AL^2}{PM^2}.$$

7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Sol. $\triangle BCL$ and $\triangle ACM$ are equilateral triangles.

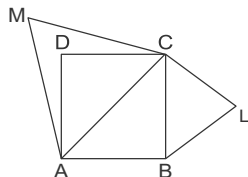
$$\triangle BCL \sim \triangle ACM \quad (\text{AAA-Similarity})$$

$$\Rightarrow \frac{ar(\triangle BCL)}{ar(\triangle ACM)} = \frac{BC^2}{AC^2} \quad \dots(i)$$

$$\text{Also } AC = \sqrt{2} BC \dots\dots\dots(ii)$$

From (i) and (ii), we get

$$\frac{ar(\triangle BCL)}{ar(\triangle ACM)} = \frac{BC^2}{(\sqrt{2}BC)^2} = \frac{1}{2}$$



$$\therefore \text{ar}(\triangle BCL) = \frac{1}{2} \text{ar}(\triangle ACM).$$

8. *ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is*

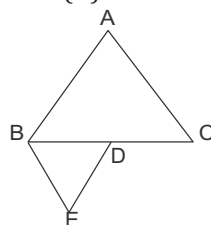
(A) 2 : 1 (B) 1 : 2 (C) 4 : 1 (D) 1 : 4

Sol. As $\triangle ABC$ and $\triangle BDE$ are equilateral.

$\therefore \triangle ABC \sim \triangle BDE$. Also $BC = 2BD$.

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle BDE)} = \frac{BC^2}{BD^2} = \frac{(2BD)^2}{BD^2} = \frac{4}{1}.$$

$\therefore$ Option (C) is correct.



9. *Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio*

(A) 2 : 3 (B) 4 : 9 (C) 81 : 16 (D) 16 : 81

Sol. $\frac{\text{Area of first triangle}}{\text{Area of second triangle}} = \left(\frac{4}{9} \right)^2 = \frac{16}{81}.$

$\therefore$ Option (D) is correct.

Exercise 6.5

1. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

- (i) 7 cm, 24 cm, 25 cm
- (ii) 3 cm, 8 cm, 6 cm
- (iii) 50 cm, 80 cm, 100 cm
- (iv) 13 cm, 12 cm, 5 cm

Sol. If square of one side is equal to the sum of the squares of the other two, then we say, “the sides form a right-angled triangle.”

(i) $(25)^2 = (7)^2 + (24)^2 \Rightarrow 625 = 49 + 576.$

Hence, the sides are of a right-angled triangle.

The length of the hypotenuse is 25 cm.

(ii) $(8)^2 \neq (3)^2 + (6)^2 \Rightarrow 64 \neq 9 + 36,$

Hence, the sides are not of a right-angled triangle.

$$(iii) \quad (100)^2 \neq (50)^2 + (80)^2 \Rightarrow 10000 \neq 2500 + 6400$$

Hence, the sides are not of a right-angled triangle.

$$(iv) \quad (13)^2 = (12)^2 + (5)^2 \Rightarrow 169 = 144 + 25$$

Hence, the sides are of a right-angled triangle.

The length of the hypotenuse is 13 cm.

2. *PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM \cdot MR$.*

Sol. Let $\angle Q = x$

$$\Rightarrow \angle MPQ = 90^\circ - x$$

$$\text{And } \angle RPM = 90^\circ - (90^\circ - x) = x$$

$$\Rightarrow \angle Q = \angle RPM$$

In $\triangle PMQ$ and $\triangle PMR$,

$$\angle Q = \angle RPM$$

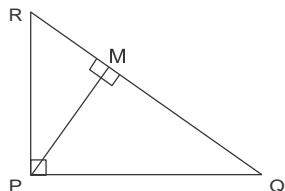
$$\angle PMQ = \angle PMR$$

$$\therefore \triangle PQM \sim \triangle RPM$$

[Proved above]

[AA similarity]

$$\Rightarrow \frac{PM}{RM} = \frac{QM}{PM} \Rightarrow PM^2 = QM \cdot MR.$$



Hence proved.

3. *In figure, ABD is a triangle right angled at A and $AC \perp BD$. Show that:*

$$(i) \quad AB^2 = BC \cdot BD$$

$$(ii) \quad AC^2 = BC \cdot DC$$

$$(iii) \quad AD^2 = BD \cdot CD$$

Sol. (i) In $\triangle ABC$ and $\triangle ABD$,

$\angle B$ is common.

$$\angle ACB = \angle BAD$$

[90° each]

$$\Rightarrow \triangle BAD \sim \triangle BCA$$

[AA similarity]

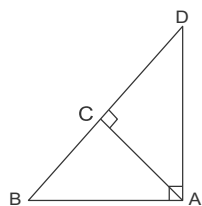
$$\therefore \frac{AB}{BD} = \frac{CB}{BA} \Rightarrow AB^2 = BC \cdot BD.$$

(ii) In $\triangle ABC$ and $\triangle ACD$

$$\angle ADC = \angle CAB$$

$$\angle ACD = \angle ACB$$

[90° each]



$$\Rightarrow \triangle ABC \sim \triangle DAC \quad [\text{AA similarity}]$$

$$\Rightarrow \frac{AC}{BC} = \frac{DC}{AC} \Rightarrow AC^2 = BC \cdot DC.$$

(iii) In $\triangle ABD$ and $\triangle CAD$,

$$\angle BAD = \angle ACD \quad [90^\circ \text{ each}]$$

and $\angle D$ is common.

$$\Rightarrow \triangle ABD \sim \triangle CAD \quad [\text{AA similarity}]$$

$$\therefore \frac{AD}{CD} = \frac{BD}{AD}$$

$$\Rightarrow AD^2 = BD \cdot CD. \quad \text{Hence proved.}$$

4. ABC is an isosceles triangle right angled at C . Prove that $AB^2 = 2AC^2$.

Sol. $\triangle ABC$ is right angled at C .

$$\Rightarrow AB^2 = AC^2 + BC^2. \quad \text{Also } AC = BC.$$

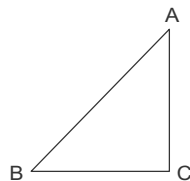
$$\therefore AB^2 = AC^2 + AC^2 = 2AC^2. \quad \text{Hence proved.}$$

5. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Sol. $AC = BC \quad \dots(i)$

$$\begin{aligned} AB^2 &= 2AC^2 = AC^2 + AC^2 \\ &= AC^2 + BC^2 \quad [\text{From (i)}] \end{aligned}$$

$$\Rightarrow \triangle ABC \text{ is right-angled at } C.$$



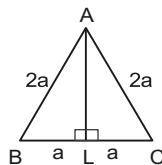
[Converse of Pythagoras theorem]

6. ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Sol. In equilateral triangle, altitude bisects the base.

$$\therefore AL = \sqrt{(2a)^2 - (a)^2} = \sqrt{4a^2 - a^2} = \sqrt{3} a.$$

Similarly, we can find out that each of other two altitudes as $\sqrt{3} a$.



7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Sol. ABCD is a rhombus and we know diagonals of a rhombus bisect each other at right angles.

$\triangle OAD$ is right-angled at O.

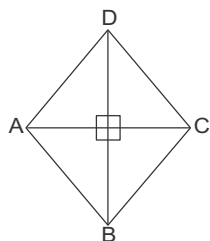
$$\therefore AD^2 = OA^2 + OD^2 \quad \dots(i)$$

Similarly,

$$AB^2 = OA^2 + OB^2 \quad \dots(ii)$$

$$BC^2 = OB^2 + OC^2 \quad \dots(iii)$$

$$CD^2 = OC^2 + OD^2 \quad \dots(iv)$$



From (i), (ii), (iii) and (iv), we get

$$AB^2 + BC^2 + CD^2 + DA^2 = 2[OA^2 + OB^2 + OC^2 + OD^2]$$

$$= 2 \left[\left(\frac{AC}{2} \right)^2 + \left(\frac{BD}{2} \right)^2 + \left(\frac{AC}{2} \right)^2 + \left(\frac{BD}{2} \right)^2 \right]$$

$$= AC^2 + BD^2.$$

8. In figure, O is a point in the interior of a triangle ABC, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$. Show that

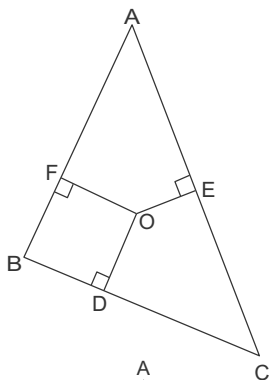
$$(i) \quad OA^2 + OB^2 + OC^2 - OD^2$$

$$- OE^2 - OF^2$$

$$= AF^2 + BD^2 + CE^2,$$

$$(ii) \quad AF^2 + BD^2 + CE^2$$

$$= AE^2 + CD^2 + BF^2.$$



Sol. (i) In right triangles AOF, BOD and COE, we have respectively

$$OA^2 = AF^2 + OF^2$$

$$OB^2 = BD^2 + OD^2$$

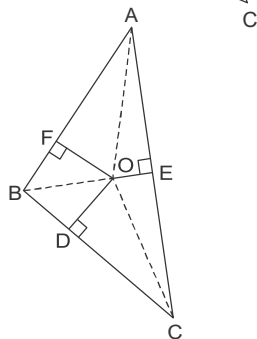
and $OC^2 = CE^2 + OE^2.$

$$\Rightarrow AF^2 + BD^2 + CE^2$$

$$= OA^2 + OB^2 + OC^2$$

$$- OF^2 - OD^2 - OE^2$$

$$(ii) \quad OB^2 - OC^2 = BD^2 - CD^2$$



$$OC^2 - OA^2 = CE^2 - AE^2$$

$$OA^2 - OB^2 = AF^2 - BF^2.$$

$$\Rightarrow 0 = BD^2 - CD^2 + CE^2 - AE^2 + AF^2 - BF^2$$

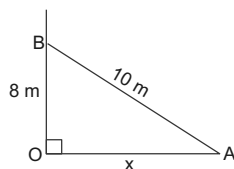
$$\Rightarrow BD^2 + CE^2 + AF^2 = CD^2 + AE^2 + BF^2.$$

9. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Sol. Window is at B.

$$AB = 10 \text{ m}, OB = 8 \text{ m}$$

$$\begin{aligned} \text{Distance } x &= \sqrt{(10)^2 - (8)^2} \text{ m} \\ &= \sqrt{36} \text{ m} = 6 \text{ m}. \end{aligned}$$



10. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Sol. AB is a pole of 18 m.

AC is a guy wire of 24 m.

$$BC = x \text{ m}$$

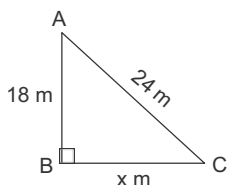
In $\triangle ABC$,

$$AC^2 = (AB)^2 + (BC)^2$$

$$(24)^2 = (18)^2 + x^2$$

$$\Rightarrow x^2 = 576 - 324 \Rightarrow x^2 = 252$$

$$\Rightarrow x = \sqrt{252} \text{ m} = 6\sqrt{7} \text{ m}.$$



11. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two

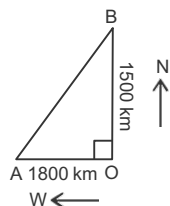
planes after $1\frac{1}{2}$ hours?

Sol. Distance OB travelled due north from the starting point O in $1\frac{1}{2}$ hours is 1500 km.

Distance OA travelled due west from the starting point

O in $1\frac{1}{2}$ hours is 1800 km.

$$\begin{aligned}\therefore AB &= \sqrt{(1500)^2 + (1800)^2} \text{ km} \\ &= 100\sqrt{225 + 324} \text{ km} \\ &= 100\sqrt{549} \text{ km} = 300\sqrt{61} \text{ km}.\end{aligned}$$



12. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Sol. Draw $AE \parallel BD$, meeting CD at E .

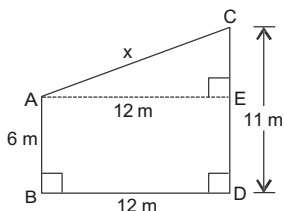
$$\therefore CE = (11 - 6) \text{ m} = 5 \text{ m}$$

$$\text{and } AE = BD = 12 \text{ m}.$$

In right-angled triangle AEC ,

$$AC^2 = AE^2 + CE^2$$

$$\Rightarrow x = \sqrt{(12)^2 + (5)^2} \text{ m} = 13 \text{ m}.$$



13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C . Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Sol. Using Pythagoras theorem in right triangles $\triangle AEC$,

$\triangle BDC$, $\triangle ABC$ and $\triangle DEC$, we have respectively.

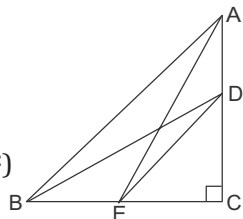
$$AE^2 = EC^2 + AC^2$$

$$BD^2 = BC^2 + CD^2$$

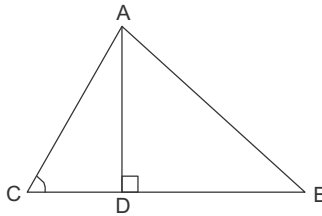
$$AB^2 = BC^2 + AC^2$$

$$\text{and } DE^2 = CD^2 + CE^2$$

$$\begin{aligned}AE^2 + BD^2 &= EC^2 + AC^2 + BC^2 + CD^2 \\ &= (EC^2 + DC^2) + (AC^2 + BC^2) \\ &= DE^2 + AB^2.\end{aligned}$$



14. The perpendicular from A on side BC of a $\triangle ABC$ intersects BC at D such that $DB = 3 CD$ (see figure). Prove that $2AB^2 = 2AC^2 + BC^2$.



Sol.

$$BD = 3 \text{ CD} \Rightarrow BD = \frac{3}{4} BC, \text{ CD} = \frac{1}{4} BC$$

$$\begin{aligned} AB^2 - AC^2 &= (BD^2 + AD^2) - (AD^2 + CD^2) \\ &= BD^2 - CD^2 = \frac{9}{16} BC^2 - \frac{1}{16} BC^2 = \frac{1}{2} BC^2 \end{aligned}$$

$$\Rightarrow 2AB^2 - 2AC^2 = BC^2.$$

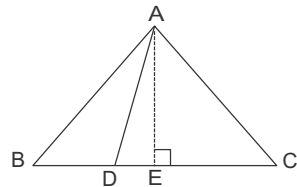
$$\therefore 2AB^2 = 2AC^2 + BC^2.$$

- 15.** In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

Sol. Given: In $\triangle ABC$, $AB = BC = CA$

D is a point on BC such that

$$BD = \frac{1}{3} BC$$



To prove: $9AD^2 = 7AB^2$

Construction: Draw $AE \perp BC$, which intersects BC at E.

Proof: As $\triangle ABC$ is an equilateral triangle.

$$\therefore BE = EC = \frac{BC}{2} = \frac{AB}{2} \quad \dots(i)$$

$$\begin{aligned} \text{Further, } DE &= BE - BD = \frac{AB}{2} - \frac{BC}{3} \\ &\quad \text{[From construction]} \end{aligned}$$

$$= \frac{AB}{2} - \frac{AB}{3} = \frac{AB}{6} \quad \dots(ii)$$

In right-angled triangle ABE,

$$AE^2 = AB^2 - BE^2 = AB^2 - \left(\frac{AB}{2}\right)^2 = \frac{3}{4} AB^2 \quad \dots(iii)$$

[From (i)]

Also, in right-angled triangle ADE,

$$AE^2 = AD^2 - DE^2 = AD^2 - \frac{AB^2}{36} \quad \dots(iv)$$

[From (ii)]

From equations (iii) and (iv), we have

$$\frac{3}{4} AB^2 = AD^2 - \frac{AB^2}{36}$$

$$\Rightarrow 7AB^2 = 9AD^2.$$

Hence proved.

- 16.** In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

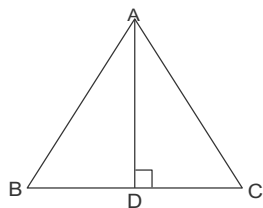
Sol. Let side of an equilateral triangle be a and AD is altitude.

In right angled triangle ADB.

$$a^2 = AD^2 + \left(\frac{a}{2}\right)^2$$

$$\Rightarrow AD^2 = \frac{3}{4} a^2$$

$$\Rightarrow 4AD^2 = 3(AB)^2.$$



- 17.** Tick the correct answer and justify: In

ΔABC , $AB = 6\sqrt{3}$ cm,

$AC = 12$ cm and $BC = 6$ cm. The angle B is:

(A) 120°

(B) 60°

(C) 90°

(D) 45°

Sol. $AB^2 = 108$, $AC^2 = 144$, $BC^2 = 36$.

As $AC^2 = AB^2 + BC^2$.

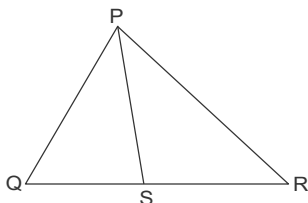
$\Rightarrow$ Triangle is right angled at B. Therefore, $\angle B = 90^\circ$.

Hence, option (C) is correct.

Exercise 6.6 (OPTIONAL)

1. In figure, PS is the bisector of $\angle QPR$ of $\triangle PQR$. Prove that

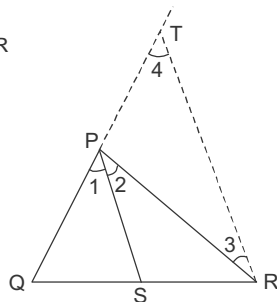
$$\frac{QS}{SR} = \frac{PQ}{PR}.$$



Sol. Given: In $\triangle PQR$, PS is bisector of $\angle QPR$, meeting QR at S.

To prove: $\frac{QS}{SR} = \frac{PQ}{PR}.$

Construction: Draw $RT \parallel PS$, meeting QP produced at T.



Proof: $\angle 1 = \angle 4$

...(i) [Corresponding angles]

$\angle 2 = \angle 3$

...(ii) [Alternate angles]

Also $\angle 1 = \angle 2$

...(iii) [$\because$ PS is bisector of $\angle QPR$]

$\Rightarrow \angle 3 = \angle 4$

[From (i), (ii), (iii)]

$\Rightarrow PR = PT$

...(iv)

In $\triangle QTR$, $PS \parallel RT$

[Construction]

$\Rightarrow \frac{QS}{SR} = \frac{QP}{PT} \Rightarrow \frac{QS}{SR} = \frac{QP}{PR}$

[From (i)]

Hence proved.

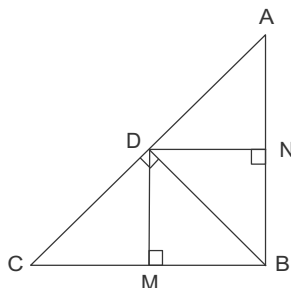
2. In figure, D is a point on hypotenuse AC of $\triangle ABC$, such that $BD \perp AC$, $DM \perp BC$ and $DN \perp AB$. Prove that:

(i) $DM^2 = DN \cdot MC$

(ii) $DN^2 = DM \cdot AN$

Sol. In quadrilateral DMBN

$\angle DMB = \angle MBN = \angle BND = 90^\circ$ each



$$\therefore \angle MDN = 90^\circ$$

$\Rightarrow$ DMBN is a rectangle

$$\Rightarrow MB = DN \text{ and } DM = NB \quad \dots(i)$$

In triangle CMD and DMB,

$$\angle CMD = \angle DMB = 90^\circ \text{ each}$$

$$\angle C = \angle MDB$$

$$\therefore \triangle CMD \sim \triangle DMB$$

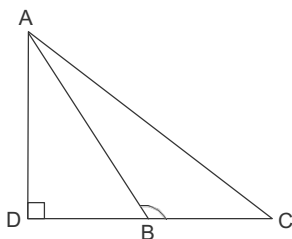
$$\Rightarrow \frac{DM}{MC} = \frac{BM}{MD} \Rightarrow DM^2 = BM \cdot MC = DN \cdot MC \quad [\text{From (i)}]$$

Similarly we can show

$$\triangle AND \sim \triangle DNB$$

$$\text{and } DN^2 = DM \cdot AN.$$

3. In figure, ABC is a triangle in which $\angle ABC > 90^\circ$ and $AD \perp CB$ produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$.



Sol. In right-angled $\triangle ADC$, $AC^2 = AD^2 + CD^2$

[Pythagoras Theorem]

$$\Rightarrow AD^2 = AC^2 - CD^2 \quad \dots(i)$$

Also, in right-angled $\triangle ADB$, $AB^2 = AD^2 + BD^2$

[Pythagoras Theorem]

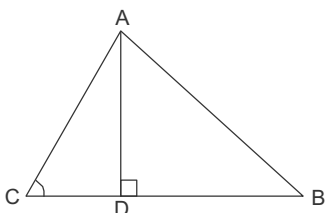
$$\Rightarrow AD^2 = AB^2 - BD^2 \quad \dots(ii)$$

From (i) and (ii), we get

$$\begin{aligned} AC^2 &= AB^2 + CD^2 - BD^2 \\ &= AB^2 + (BC + BD)^2 - BD^2 \\ &= AB^2 + BC^2 + 2BC \cdot BD + BD^2 - BD^2 \\ &= AB^2 + BC^2 + 2BC \cdot BD. \end{aligned}$$

Hence proved.

4. In figure, ABC is a triangle in which $\angle ABC < 90^\circ$ and $AD \perp BC$.
Prove that $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$.



Sol. Consider right-angled triangle ADB.

$$AB^2 = AD^2 + BD^2 \quad \dots(i) \quad [\text{Pythagoras Theorem}]$$

and in right-angled triangle ADC,

$$AC^2 = AD^2 + DC^2 \quad \dots(ii) \quad [\text{Pythagoras Theorem}]$$

$$\Rightarrow AC^2 - AB^2 = DC^2 - BD^2 \quad [\text{From (i) and (ii)}]$$

$$\Rightarrow AC^2 = AB^2 + (BC - BD)^2 - BD^2$$

$$\Rightarrow AC^2 = AB^2 + BC^2 + BD^2 - 2BC \cdot BD - BD^2$$

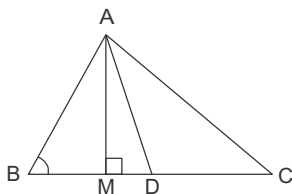
$$\Rightarrow AC^2 = AB^2 + BC^2 - 2BC \cdot BD.$$

5. In figure, AD is a median of a triangle ABC and $AM \perp BC$.
Prove that:

$$(i) \quad AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

$$(ii) \quad AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

$$(iii) \quad AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$$



Sol. Consider right-angled triangle AMC.

$$(i) \quad AC^2 = AM^2 + MC^2 \quad \dots(i) \quad [\text{Pythagoras Theorem}]$$

Also in right-angled-triangle AMD,

$$AD^2 = AM^2 + MD^2 \quad \dots(ii) \quad [\text{Pythagoras Theorem}]$$

$$\Rightarrow AC^2 - AD^2 = MC^2 - MD^2 \quad [\text{From (i), (ii)}]$$

$$= (MC + MD)(MC - MD)$$

$$= (DC + MD + MD)(DC)$$

$$= DC^2 + 2MD \cdot DC$$

$$= \left(\frac{BC}{2} \right)^2 + 2MD \cdot \frac{BC}{2} \quad [\because D \text{ is mid-point of } BC]$$

$$\Rightarrow AC^2 = AD^2 + MD \cdot BC + \left(\frac{BC}{2} \right)^2$$

(ii) Consider right-angled triangle AMD.

$$AB^2 = AM^2 + BM^2 \quad \dots(i) \text{ [Pythagoras Theorem]}$$

Also in right-angled triangle AMD,

$$AD^2 = AM^2 + MD^2 \quad \dots(ii) \text{ [Pythagoras Theorem]}$$

$$\Rightarrow AB^2 - AD^2 = BM^2 - MD^2 \quad \text{[From (i) and (ii)]}$$

$$= (BM + MD)(BM - MD)$$

$$= (BD)(BD - MD - MD)$$

$$\Rightarrow AB^2 = AD^2 + BD^2 - 2BD \cdot MD$$

$$= AD^2 + \left(\frac{BC}{2} \right)^2 - 2 \cdot \frac{BC}{2} \cdot MD$$

$[\because D \text{ is mid-point of } BC]$

$$\Rightarrow AB^2 = AD^2 + \left(\frac{BC}{2} \right)^2 - BC \cdot DM.$$

(iii) Adding result (i) and (ii), we get

$$AC^2 + AB^2 = 2AD^2 + \frac{1}{2} (BC)^2.$$

6. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Sol. Draw $CL \perp AB$ and $DM \perp AB$.

In triangles AMD and BLC,

$$AD = BC \quad \text{[Opposite sides of a ||gm]}$$

$$DM = CL \quad \text{[Distance between two parallel lines]}$$

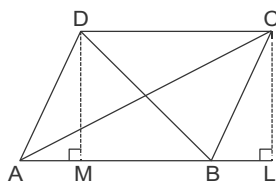
$$\angle DMA = \angle CLB = [90^\circ \text{ each}]$$

$$\therefore \triangle AMD \cong \triangle BLC$$

[RHS similarity]

$$\Rightarrow AM = BL \quad \dots(i)$$

In right-angled triangle ALC,



$$AC^2 = AL^2 + CL^2 \quad (ii)$$

In right-angled triangle BLC,

$$BC^2 = BL^2 + CL^2 \quad \dots(iii)$$

$$\Rightarrow AC^2 - BC^2 = AL^2 - BL^2 \quad [\text{From (ii) and (iii)}]$$

$$= (AL + BL)(AL - BL)$$

$$= (AB + BL + BL)(AB)$$

$$= AB^2 + 2BL.AB$$

$$\Rightarrow AC^2 = BC^2 + AB^2 + 2BL.AB \quad \dots(iv)$$

In right-angled triangle DMB,

$$DB^2 = DM^2 + MB^2 \quad \dots(v)$$

and in right-angled triangle DMA,

$$AD^2 = DM^2 + AM^2 \quad \dots(vi)$$

$$\Rightarrow DB^2 - AD^2 = MB^2 - AM^2 \quad \dots[\text{From (v) and (vi)}]$$

$$= (MB - AM)(MB + AM)$$

$$= (AB - AM - AM)(AB)$$

$$= AB^2 - 2AM.AB$$

$$\Rightarrow BD^2 = AD^2 + AB^2 - 2AM.AB \quad \dots(vii)$$

$$\therefore AC^2 + BD^2 = BC^2 + AB^2 + AD^2 + AB^2$$

[From (iv), (vii) and (i)]

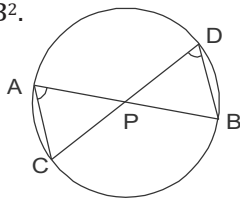
$$\Rightarrow AC^2 + BD^2 = BC^2 + DC^2 + AD^2 + AB^2.$$

7.

In figure, two chords AB and CD intersect each other at the point P. Prove that:

$$(i) \triangle APC \sim \triangle DPB$$

$$(ii) AP \cdot PB = CP \cdot DP$$



Sol. Given: Chords AB and CD of given circle intersect at P.

To prove: $\triangle APC \sim \triangle DPB$.

Proof: $\angle 1 = \angle 2$

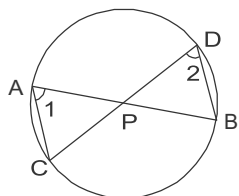
[Angles in the same segment]

$$\angle APC = \angle DPB$$

[Vertically opposite angles]

$$(i) \therefore \triangle APC \sim \triangle DPB$$

$$(ii) \frac{AP}{PC} = \frac{DP}{PB}$$



[AA similarity]

[Corresponding sides]

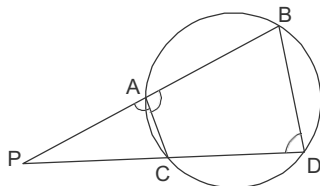
$$\Rightarrow AP \cdot PB = CP \cdot DP.$$

Hence proved.

8. In figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that:

(i) $\triangle PAC \sim \triangle PDB$

(ii) $PA \cdot PB = PC \cdot PD$



Sol. Given: Chords AB and CD intersect at P outside the circle.

Proof: $\angle 1 + \angle 2 = 180^\circ$... (a) [Linear pair]

$\angle 2 + \angle 3 = 180^\circ$... (b)

[Sum of pair of opposite angles of a cyclic quadrilateral]

From (a) and (b), we get

$$\angle 1 + \angle 2 = \angle 2 + \angle 3 \Rightarrow \angle 1 = \angle 3$$

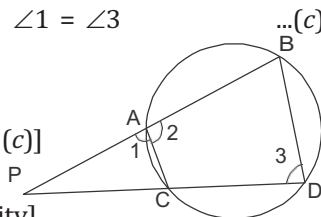
- (i) In $\triangle PAC$ and $\triangle PBD$,

$\angle P$ is common.

$\therefore \angle 1 = \angle 3$ [From (c)]

$\therefore \triangle PAC \sim \triangle PBD$

[AA similarity]



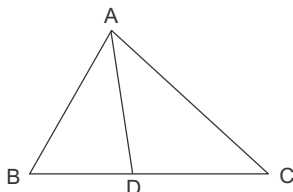
(ii) $\frac{AP}{CP} = \frac{DP}{BP}$

[Corresponding sides]

$\Rightarrow AP \cdot PB = CP \cdot DP.$

Hence proved.

9. In figure, D is a point on side BC of $\triangle ABC$ such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of $\angle BAC$.



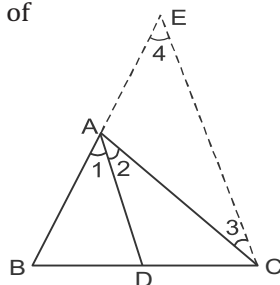
Sol. Given: D is a point on the side BC of

a $\triangle ABC$ such that $\frac{BD}{DC} = \frac{AB}{AC}$.

To prove: AD is bisector of $\angle BAC$

Construction: Draw $CE \parallel DA$, meeting BA produced at E.

Proof: In $\triangle BEC$, $AD \parallel CE$



$$\Rightarrow \frac{BD}{DC} = \frac{AB}{AE} \quad \dots(i) \text{ [Basic Proportionality Theorem]}$$

$$\text{Also } \frac{BD}{DC} = \frac{AB}{AC} \quad \dots(ii) \text{ [Given]}$$

$$\Rightarrow \frac{AB}{AE} = \frac{AB}{AC} \quad \text{[From (i) and (ii)]}$$

$$\Rightarrow AE = AC$$

In $\triangle ACE$, as $AE = AC$ [Proved above]

$$\Rightarrow \angle 3 = \angle 4 \quad \dots(iii)$$

$$\text{Also } \angle 3 = \angle 2 \quad \dots(iv) \text{ [Alternate angles]}$$

$$\text{and } \angle 1 = \angle 4 \quad \dots(v) \text{ [Corresponding angles]}$$

From (iii), (iv) and (v), we get

$$\angle 1 = \angle 2$$

$\Rightarrow$ AD is the bisector of $\angle BAC$.

- 10.** Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?

Sol. $BO = \sqrt{(2.4)^2 + (1.8)^2} \text{ m}$

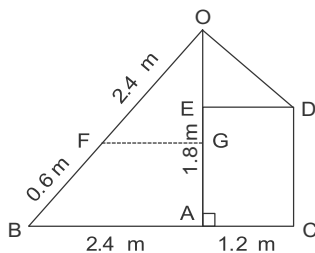
$$= \sqrt{5.76 + 3.24} \text{ m} = \sqrt{9.00} = 3 \text{ m}$$

After 12 second, let string is at F,

Distance BF covered in 12 seconds

$$= 5 \times 12 \text{ cm} = 60 \text{ cm} = 0.6 \text{ m}$$

$$\therefore OF = BO - BF = 3 - 0.6 = 2.4 \text{ m}$$



As $\triangle OBA \sim \triangle OFG$,

$$\text{We have } \frac{OB}{OF} = \frac{AB}{FG} \Rightarrow \frac{3}{2.4} = \frac{2.4}{FG}$$

$$\Rightarrow FG = \frac{5.76}{3} = 1.92 \text{ m}$$

$\therefore$ Horizontal distance of the fly

$$= FG + AC$$

$$= (1.92 + 1.2) \text{ m} = 3.12 \text{ m}.$$