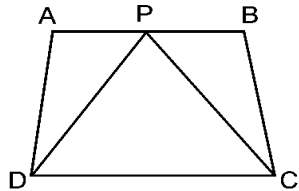


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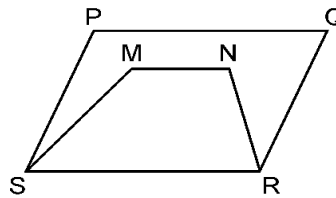
AREAS OF PARALLELOGRAMS AND TRIANGLES

EXERCISE 9.1

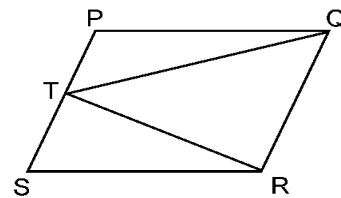
Q.1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.



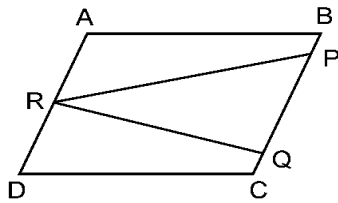
(i)



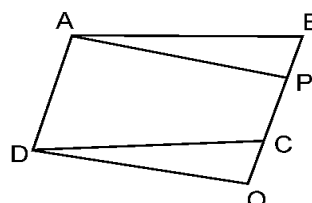
(ii)



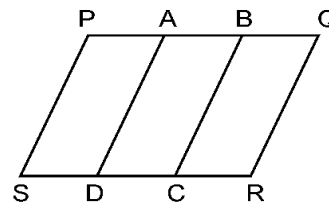
(iii)



(iv)



(v)



(vi)

Sol. (i) Base DC, parallels DC and AB
 (iii) Base QR, parallels QR and PS
 (v) Base AD, parallels AD and BQ.

9

AREAS OF PARALLELOGRAMS AND TRIANGLES

EXERCISE 9.2

Q.1. In the figure, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm, find AD .

Sol. Area of parallelogram $ABCD$

$$= AB \times AE$$

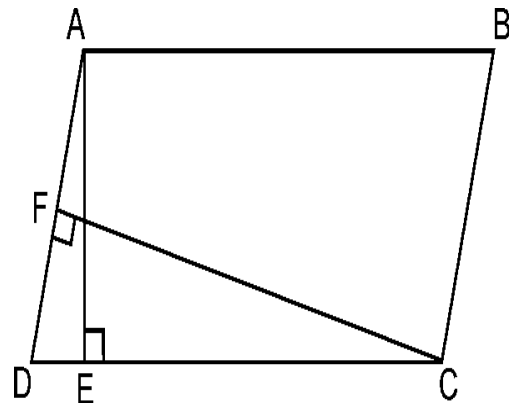
$$= 16 \times 8 \text{ cm}^2 = 128 \text{ cm}^2$$

Also, area of parallelogram $ABCD$

$$= AD \times FC = (AD \times 10) \text{ cm}^2$$

$$\therefore AD \times 10 = 128$$

$$\Rightarrow AD = \frac{128}{10} = 12.8 \text{ cm Ans.}$$



Q.2. If E , F , G , and H are respectively the mid-points of the sides of a parallelogram $ABCD$, show that $\text{ar}(EFGH) = \frac{1}{2} \text{ar}(ABCD)$.

Sol. Given : A parallelogram $ABCD$ · E , F , G , H are mid-points of sides AB , BC , CD , DA respectively

To Prove : $\text{ar}(EFGH) = \frac{1}{2} \text{ar}(ABCD)$

Construction : Join AC and HF.

Proof : In $\triangle ABC$,

E is the mid-point of AB.

F is the mid-point of BC.

$$\Rightarrow EF \text{ is parallel to AC and } EF = \frac{1}{2} AC \dots (i)$$

Similarly, in $\triangle ADC$, we can show that

$$HG \parallel AC \text{ and } HG = \frac{1}{2} AC \dots (ii)$$

From (i) and (ii)

$EF \parallel HG$ and $EF = HG$

$\therefore$ EFGH is a parallelogram.

[One pair of opposite sides is equal and parallel]

In quadrilateral ABFH, we have

$$HA = FB \text{ and } HA \parallel FB \quad [AD = BC \Rightarrow \frac{1}{2} AD = \frac{1}{2} BC \Rightarrow HA = FB]$$

$\therefore$ ABFH is a parallelogram.

[One pair of opposite sides is equal and parallel]

Now, triangle HEF and parallelogram HABF are on the same base HF and between the same parallels HF and AB.

$$\therefore \text{Area of } \triangle HEF = \frac{1}{2} \text{ area of HABF} \dots (iii)$$

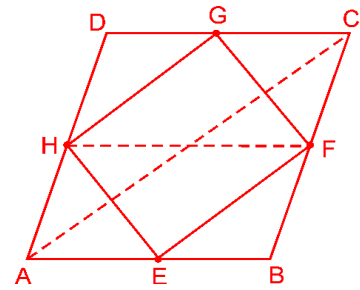
$$\text{Similarly, area of } \triangle HGF = \frac{1}{2} \text{ area of HFCD} \dots (iv)$$

Adding (iii) and (iv),

Area of $\triangle HEF$ + area of $\triangle HGF$

$$= \frac{1}{2} (\text{area of HABF} + \text{area of HFCD})$$

$$\Rightarrow \text{ar (EFGH)} = \frac{1}{2} \text{ar (ABCD)} \text{ **Proved.**}$$



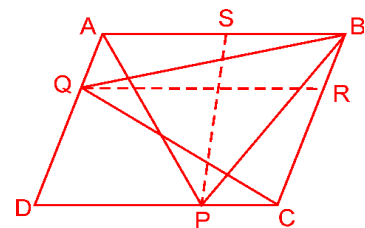
Q.3. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that $\text{ar (APB)} = \text{ar (BQC)}$.

Sol. Given : A parallelogram ABCD. P and Q are any points on DC and AD respectively.

To prove : $\text{ar (APB)} = \text{ar (BQC)}$

Construction : Draw $PS \parallel AD$ and $QR \parallel AB$.

Proof : In parallelogram ABRQ, BQ is the diagonal.



$$\therefore \text{area of } \triangle BQR = \frac{1}{2} \text{ area of ABRQ} \dots (i)$$

In parallelogram CDQR, CQ is a diagonal.

$$\therefore \text{area of } \triangle RQC = \frac{1}{2} \text{ area of } CDQR \quad \dots (ii)$$

Adding (i) and (ii), we have
area of $\triangle BQR$ + area of $\triangle RQC$

$$= \frac{1}{2} [\text{area of } ABRQ + \text{area of } CDQR]$$

$$\Rightarrow \text{area of } \triangle BQC = \frac{1}{2} \text{ area of } ABCD \quad \dots (iii)$$

Again, in parallelogram DPSA, AP is a diagonal.

$$\therefore \text{area of } \triangle ASP = \frac{1}{2} \text{ area of } DPSA \quad \dots (iv)$$

In parallelogram BCPS, PB is a diagonal.

$$\therefore \text{area of } \triangle BPS = \frac{1}{2} \text{ area of } BCPS \quad \dots (v)$$

Adding (iv) and (v)

$$\text{area of } \triangle ASP + \text{area of } \triangle BPS = \frac{1}{2} (\text{area of } DPSA + \text{area of } BCPS)$$

$$\Rightarrow \text{area of } \triangle APB = \frac{1}{2} (\text{area of } ABCD) \quad \dots (vi)$$

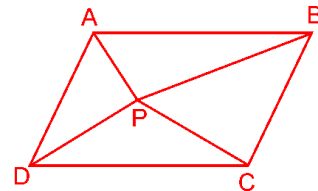
From (iii) and (vi), we have

area of $\triangle APB$ = area of $\triangle BQC$. **Proved.**

Q.4. In the figure, P is a point in the interior of a parallelogram ABCD. Show that

$$(i) \text{ ar } (APB) + \text{ ar } (PCD) = \frac{1}{2} \text{ ar } (ABCD)$$

$$(ii) \text{ ar } (APD) + \text{ ar } (PBC) = \text{ ar } (APB) + \text{ ar } (PCD)$$

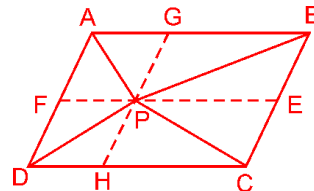


Sol. Given : A parallelogram ABCD. P is a point inside it.

To prove : (i) $\text{ ar } (APB) + \text{ ar } (PCD)$

$$= \frac{1}{2} \text{ ar } (ABCD)$$

$$(ii) \text{ ar } (APD) + \text{ ar } (PBC) \\ = \text{ ar } (APB) + \text{ ar } (PCD)$$



Construction : Draw EF through P parallel to AB, and GH through P parallel to AD.

Proof : In parallelogram FPGA, AP is a diagonal,

$$\therefore \text{area of } \triangle APG = \text{area of } \triangle APF \quad \dots (i)$$

In parallelogram BGPE, PB is a diagonal,

$$\therefore \text{area of } \triangle BPG = \text{area of } \triangle EPB \quad \dots (ii)$$

In parallelogram DHPF, DP is a diagonal,

$$\therefore \text{area of } \triangle DPH = \text{area of } \triangle DPF \quad \dots \text{ (iii)}$$

In parallelogram HCEP, CP is a diagonal,

$$\therefore \text{area of } \triangle CPH = \text{area of } \triangle CPE \quad \dots \text{ (iv)}$$

Adding (i), (ii), (iii) and (iv)

$$\begin{aligned} & \text{area of } \triangle APG + \text{area of } \triangle BPG + \text{area of } \triangle DPH + \text{area of } \triangle CPH \\ &= \text{area of } \triangle APF + \text{area of } \triangle EPB + \text{area of } \triangle DPF + \text{area of } \triangle CPE \\ &\Rightarrow [\text{area of } \triangle APG + \text{area of } \triangle BPG] + [\text{area of } \triangle DPH + \text{area of } \triangle CPH] \\ &= [\text{area of } \triangle APF + \text{area of } \triangle DPF] + [\text{area of } \triangle EPB + \text{area of } \triangle CPE] \\ &\Rightarrow \text{area of } \triangle APB + \text{area of } \triangle CPD = \text{area of } \triangle APD + \text{area of } \triangle BPC \\ &\quad \dots \text{ (v)} \end{aligned}$$

But area of parallelogram ABCD

$$= \text{area of } \triangle APB + \text{area of } \triangle CPD + \text{area of } \triangle APD + \text{area of } \triangle BPC \quad \dots \text{ (vi)}$$

From (v) and (vi)

$$\text{area of } \triangle APB + \text{area of } \triangle PCD = \frac{1}{2} \text{ area of } ABCD$$

$$\text{or, ar (APB) + ar (PCD) = } \frac{1}{2} \text{ ar (ABCD) Proved.}$$

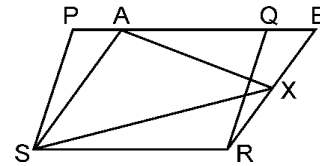
(ii) From (v),

$$\Rightarrow \text{ar (APD) + ar (PBC) = ar (APB) + ar (CPD) Proved.}$$

Q.5. In the figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

$$(i) \text{ ar (PQRS) = ar (ABRS)}$$

$$(ii) \text{ ar (AXS) = } \frac{1}{2} \text{ ar (PQRS)}$$



Sol. Given : PQRS and ABRS are parallelograms and X is any point on side BR.

To prove : (i) ar (PQRS) = ar (ABRS)

$$(ii) \text{ ar (AXS) = } \frac{1}{2} \text{ ar (PQRS)}$$

Proof : (i) In $\triangle ASP$ and $\triangle BRQ$, we have

$$\angle SPA = \angle RQB \quad [\text{Corresponding angles}] \quad \dots (1)$$

$$\angle PAS = \angle QBR \quad [\text{Corresponding angles}] \quad \dots (2)$$

$$\therefore \angle PSA = \angle QRB \quad [\text{Angle sum property of a triangle}] \quad \dots (3)$$

$$\text{Also, PS = QR} \quad [\text{Opposite sides of the parallelogram PQRS}] \quad \dots (4)$$

$$\text{So, } \triangle ASP \cong \triangle BRQ \quad [\text{ASA axiom, using (1), (3) and (4)}]$$

Therefore, area of $\triangle PSA$ = area of $\triangle QRB$

[Congruent figures have equal areas] $\dots (5)$

$$\text{Now, ar (PQRS) = ar (PSA) + ar (ASRQ)}$$

$$= \text{ar (QRB) + ar (ASRQ)}$$

$$= \text{ar (ABRS)}$$

$$\text{So, ar (PQRS) = ar (ABRS) Proved.}$$

(ii) Now, $\triangle AXS$ and $\square ABRS$ are on the same base AS and between same parallels AS and BR

$$\therefore \text{area of } \triangle AXS = \frac{1}{2} \text{ area of } ABR$$

$$\Rightarrow \text{area of } \triangle AXS = \frac{1}{2} \text{ area of } PQRS \quad [\because \text{ar } (PQRS) = \text{ar } (ABRS)]$$

$$\Rightarrow \text{ar of } (\triangle AXS) = \frac{1}{2} \text{ ar of } (PQRS) \text{ **Proved.**}$$

Q.6. A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Sol. The field is divided in three triangles.

Since triangle APQ and parallelogram PQRS are on the same base PQ and between the same parallels PQ and RS.

$$\therefore \text{ar } (\triangle APQ) = \frac{1}{2} \text{ar } (PQRS)$$

$$\Rightarrow 2\text{ar } (\triangle APQ) = \text{ar}(PQRS)$$

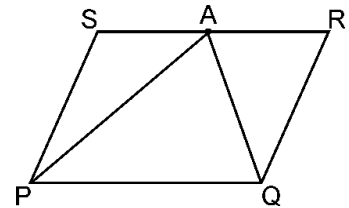
$$\text{But ar } (PQRS) = \text{ar}(\triangle APQ) + \text{ar } (\triangle PSA) + \text{ar } (\triangle ARQ)$$

$$\Rightarrow 2 \text{ ar } (\triangle APQ) = \text{ar}(\triangle APQ) + \text{ar}(\triangle PSA) + \text{ar } (\triangle ARQ)$$

$$\Rightarrow \text{ar } (\triangle APQ) = \text{ar}(\triangle PSA) + \text{ar}(\triangle ARQ)$$

Hence, area of $\triangle APQ$ = area of $\triangle PSA$ + area of $\triangle ARQ$.

To sow wheat and pulses in equal portions of the field separately, farmer sow wheat in $\triangle APQ$ and pulses in other two triangles or pulses in $\triangle APQ$ and wheat in other two triangles. **Ans.**



Mathematics

(Chapter – 9)(Areas of Parallelograms and Triangles)

(Class – 9)

Exercise 9.3

Question 1:

In Figure, E is any point on median AD of a $\triangle ABC$. Show that $ar(ABE) = ar(ACE)$.

Answer 1:

In $\triangle ABC$, AD is median. [$\because$ Given]

Hence, $ar(ABD) = ar(ACD)$... (1)

[$\because$ A median of a triangle divides it into two triangles of equal areas.]

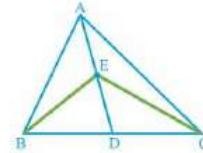
Similarly, in $\triangle EBC$, ED is median. [$\because$ Given]

Hence, $ar(EBD) = ar(ECD)$... (2)

Subtracting equation (2) from (1), we get

$$ar(ABD) - ar(EBD) = ar(ACD) - ar(ECD)$$

$$\Rightarrow ar(ABE) = ar(ACE)$$



Question 2:

In a triangle ABC, E is the mid-point of median AD. Show that $ar(BED) = \frac{1}{4}ar(ABC)$.

Answer 2:

In $\triangle ABC$, AD is median. [$\because$ Given]

Hence, $ar(ABD) = ar(ACD)$

$$\Rightarrow ar(ABD) = \frac{1}{2}ar(ABC) \quad \dots (1)$$

[$\because$ A median of a triangle divides it into two triangles of equal areas.]

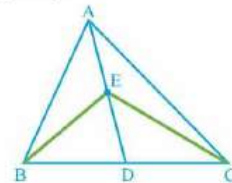
Similarly, in $\triangle ABD$, BE is median. [$\because$ E is the mid-point of AD]

Hence, $ar(BED) = ar(ABE)$

$$\Rightarrow ar(BED) = \frac{1}{2}ar(ABD)$$

$$\Rightarrow ar(BED) = \frac{1}{2} \left[\frac{1}{2}ar(ABC) \right] \quad \left[\because ar(ABD) = \frac{1}{2}ar(ABC) \right]$$

$$\Rightarrow ar(BED) = \frac{1}{4}ar(ABC)$$



Question 3:

Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Answer 3:

Diagonals of parallelogram bisect each other.

Therefore, $PO = OR$ and $SO = OQ$

In $\triangle PQS$, PO is median. [$\because$ $SO = OQ$]

Hence, $ar(PSO) = ar(PQO)$... (1)

[$\because$ A median of a triangle divides it into two triangles of equal areas.]

Similarly, in $\triangle PQR$, QO is median. [$\because$ $PO = OR$]

Hence, $ar(PQO) = ar(QRO)$... (2)

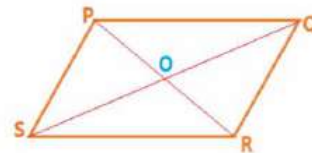
And in $\triangle QRS$, RO is median. [$\because$ $SO = OQ$]

Hence, $ar(QRO) = ar(RSO)$... (3)

From the equations (1), (2) and (3), we get

$$ar(PSO) = ar(PQO) = ar(QRO) = ar(RSO)$$

Hence, in parallelogram PQRS, diagonals PR and QS divide it into four triangles in equal area.



Question 4:

In Figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that $ar(ABC) = ar(ABD)$.

Answer 4:

In $\triangle ADC$, AO is median. $[\because CO = OD]$

Hence, $ar(ACO) = ar(ADO)$... (1)

$[\because$ A median of a triangle divides it into two triangles of equal areas.]

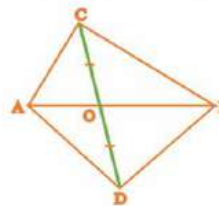
Similarly, in $\triangle BDC$, BO is median. $[\because CO = OD]$

Hence, $ar(BCO) = ar(BDO)$... (2)

Adding equation (1) and (2), we get

$$ar(ACO) + ar(BCO) = ar(ADO) + ar(BDO)$$

$$\Rightarrow ar(ABC) = ar(ABD)$$

**Question 5:**

D, E and F are respectively the mid-points of the sides BC, CA and AB of a $\triangle ABC$. Show that

(i) BDEF is a parallelogram. (ii) $ar(DEF) = \frac{1}{4}ar(ABC)$ (iii) $ar(BDEF) = \frac{1}{2}ar(ABC)$

Answer 5:

(i) In $\triangle ABC$, E and D are mid-points of CA and BC respectively

Hence, $ED \parallel AB$ and $ED = \frac{1}{2}AB$ $[\because$ Mid-point theorem]

$\Rightarrow ED \parallel AB$ and $ED = FB$ $[\because$ F is mid-point of AB]

$\Rightarrow$ BDEF is a parallelogram.

(ii) BDEF is a parallelogram. $[\because$ Proved above]

$$ar(DEF) = ar(BDF) \quad \dots (1)$$

$[\because$ Diagonal of a parallelogram divide it into two triangles, equal in area]

Similarly,

AEDF is a parallelogram.

$$ar(DEF) = ar(AEF) \quad \dots (2)$$

Similarly AEDF is a parallelogram.

$$ar(DEF) = ar(CDE) \quad \dots (3)$$

From the equation (1), (2) and (3), we get

$$ar(DEF) = ar(BDF) = ar(AEF) = ar(CDE)$$

$$\text{Let } ar(DEF) = ar(BDF) = ar(AEF) = ar(CDE) = x$$

$$\text{Therefore, } ar(ABC) = ar(DEF) + ar(BDF) + ar(AEF) + ar(CDE)$$

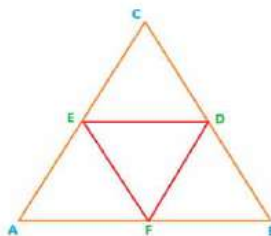
$$\Rightarrow ar(ABC) = x + x + x + x = 4x = 4ar(DEF)$$

$$\Rightarrow ar(DEF) = \frac{1}{4}ar(ABC)$$

$$(iii) ar(BDEF) = ar(DEF) + ar(BDF) = x + x = 2x$$

$$\Rightarrow ar(BDEF) = \frac{1}{2} \times 4x$$

$$\Rightarrow ar(BDEF) = \frac{1}{2} \times ar(ABC) \quad [\because ar(ABC) = 4x]$$



Question 6:

In Figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that $OB = OD$. If $AB = CD$, then show that:

- (i) $ar(DOC) = ar(AOB)$ (ii) $ar(DCB) = ar(ACB)$ (iii) $DA \parallel CB$ or ABCD is a parallelogram.

[Hint: From D and B, draw perpendiculars to AC.]

Answer 6:

(i) **Construction:** Draw perpendiculars DM and BN from D and B respectively to AC.

In $\triangle DMO$ and $\triangle BNO$,

$$\angle DMO = \angle BNO \quad [\because \text{Each } 90^\circ]$$

$$\angle DOM = \angle BON \quad [\because \text{Vertically opposite angles}]$$

$$DO = BO \quad [\because \text{Given}]$$

$$\text{Hence, } \triangle DMO \cong \triangle BNO \quad [\because \text{AAS Congruency rule}]$$

$$DM = BN \quad \dots (1) \quad [\because \text{CPCT}]$$

$$\text{And } ar(DMO) = ar(BNO) \quad \dots (2) \quad [\because \text{CPCT}]$$

In $\triangle DMC$ and $\triangle BNA$,

$$\angle DMC = \angle BNA \quad [\because \text{Each } 90^\circ]$$

$$DM = BN \quad [\because \text{From the equation (1)}]$$

$$CD = AB \quad [\because \text{Given}]$$

$$\text{Hence, } \triangle DMC \cong \triangle BNA \quad [\because \text{RHS Congruency rule}]$$

$$\text{And } ar(DMC) = ar(BNA) \quad \dots (3) \quad [\because \text{Congruent triangles area equal in area}]$$

Adding the equation (2) and (3), we get

$$ar(DMO) + ar(DMC) = ar(BNO) + ar(BNA)$$

$$\Rightarrow ar(DOC) = ar(AOB)$$

$$(ii) \quad ar(DOC) = ar(AOB) \quad [\because \text{Proved above}]$$

Adding $ar(BOC)$ both sides

$$ar(DOC) + ar(BOC) = ar(AOB) + ar(BOC)$$

$$\Rightarrow ar(DCB) = ar(ACB)$$

$$(iii) \quad \triangle DMC \cong \triangle BNA \quad [\because \text{Proved above}]$$

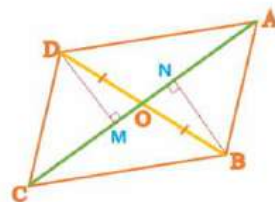
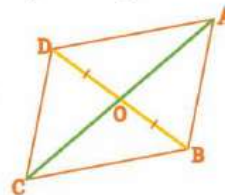
$$\angle DCM = \angle BAN \quad [\because \text{CPCT}]$$

Here, the alternate angles ($\angle DCM = \angle BAN$) are equal, Hence,

$$CD \parallel AB$$

$$\text{And } CD = AB \quad [\because \text{Given}]$$

Therefore, ABCD is a parallelogram.



Question 7:

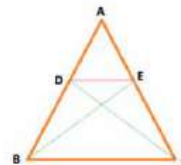
D and E are points on sides AB and AC respectively of $\triangle ABC$ such that $ar(DBC) = ar(EBC)$. Prove that $DE \parallel BC$.

Answer 7:

$\triangle DBC$ and $\triangle EBC$ are on the same base BC and $ar(DBC) = ar(EBC)$.

Therefore, $DE \parallel BC$

[$\because$ Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]



Question 8:

XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that: $ar(ABE) = ar(ACF)$

Answer 8:

In quadrilateral BCYE, BE || CY $[\because BE \parallel AC]$

BC || EY $[\because BC \parallel XY]$

Therefore, BCYE is a parallelogram.

Triangle ABE and parallelogram BCYE are on the same base BE and between same parallels, BE || AC.

Hence, $ar(ABE) = \frac{1}{2} ar(BCYE)$... (1)

$[\because \text{If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.}]$

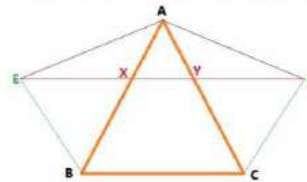
Similarly, triangle ACF and parallelogram BCFX are on the same base CF and between same parallels CF || AB.

Hence, $ar(ACF) = \frac{1}{2} ar(BCFX)$... (2)

And, $ar(BCYE) = ar(BCFX)$... (3)

$[\because \text{On the same base (BC) and between same parallels (BC || EF), area of parallelograms are equal}]$

From the equation (1), (2) and (3), $ar(ABE) = ar(ACF)$

**Question 9:**

The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see Figure). Show that $ar(ABCD) = ar(PBQR)$.

[Hint: Join AC and PQ. Now compare $ar(ACQ)$ and $ar(APQ)$.]

Answer 9:

Construction: Join AC and PQ.

Triangles ACQ and APQ lie on the same base AQ and between same parallels, AQ || CP.

Hence, $ar(ACQ) = ar(APQ)$

$[\because \text{Triangles on the same base (or equal) and between the same parallels are equal in area.}]$

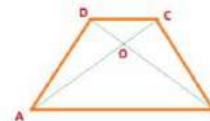
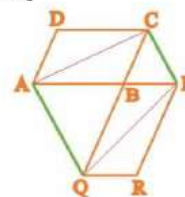
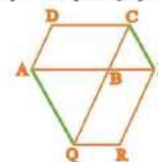
Subtracting $ar(ABQ)$ from both the sides

$$ar(ACQ) - ar(ABQ) = ar(APQ) - ar(ABQ)$$

$$\Rightarrow ar(ABC) = ar(PBQ) \Rightarrow \frac{1}{2} ar(ABCD) = \frac{1}{2} ar(PBQR)$$

$[\because \text{Diagonal divides the parallelogram in two triangles equal in area}]$

$$\Rightarrow ar(ABCD) = ar(PBQR)$$

**Question 10:**

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.

Prove that $ar(AOD) = ar(BOC)$.

Answer 10:

Triangles ABD and ABC are on the same base AB and between same parallels, AB || CD.

Hence, $ar(ABD) = ar(ABC)$

$[\because \text{Triangles on the same base (or equal bases) and between the same parallels are equal in area.}]$

Subtracting $ar(ABO)$ from both the sides

$$ar(ABD) - ar(ABO) = ar(ABC) - ar(ABO)$$

$$\Rightarrow ar(AOD) = ar(BOC)$$

Question 11:

In Figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) $ar(ACB) = ar(ACF)$

(ii) $ar(AEDF) = ar(ABCDE)$

Answer 11:

(i) Triangles ACB and ACF are on the same base AC and between same parallels AC || FB. Hence, $ar(ACB) = ar(ACF)$

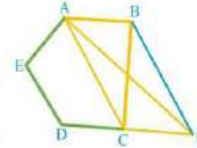
[$\because$ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]

(ii) $ar(ACB) = ar(ACF)$ [$\because$ Proved above]

Adding $ar(AEDC)$ both the sides

$$ar(ACB) + ar(AEDC) = ar(ACF) + ar(AEDC)$$

$$\Rightarrow ar(ABCDE) = ar(AEDF)$$

**Question 12:**

A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Answer 12:

Let ABCD be the Itwaari's plot.

Join BD and through C draw a line CF parallel to BD which meet AB produced at F.

Now join D and F.

Triangles CBD and FBD are on the same base BD and between same parallels BD || CF.

Hence, $ar(CBD) = ar(FBD)$

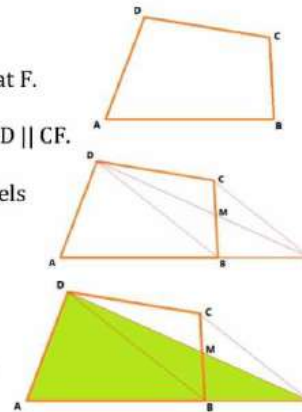
[$\because$ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]

Subtracting $ar(BDM)$ from both the sides

$$ar(CBD) - ar(BDM) = ar(FBD) - ar(BDM)$$

$$\Rightarrow ar(CMD) = ar(BFM)$$

Hence, in place of ΔCMD , if ΔBFM be given to Itwaari, his plot become triangular (ΔADF).

**Question 13:**

ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y.

Prove that $ar(ADX) = ar(ACY)$. [Hint: Join CX.]

Answer 13:

Construction: Join CX.

Triangles ADX and ACX are on the same base AX and between same parallels AB || DC.

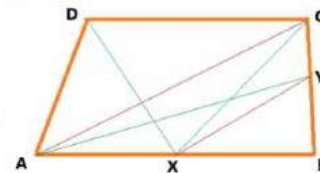
$$\text{Hence, } ar(ADX) = ar(ACX) \quad \dots (1)$$

[$\because$ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]

Similarly, triangles ACY and ACX are on the same base AC and between same parallels AC || XY.

$$\text{Hence, } ar(ACY) = ar(ACX) \quad \dots (2)$$

From the equation (1) and (2), $ar(ADX) = ar(ACY)$



Question 14:

In Figure, $AP \parallel BQ \parallel CR$. Prove that $ar(AQC) = ar(PBR)$.

Answer 14:

Triangles ABQ and PBQ are on the same base BQ and between same parallels $BQ \parallel AP$.
Hence, $ar(ABQ) = ar(PBQ)$... (1)

[$\because$ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]

Similarly,

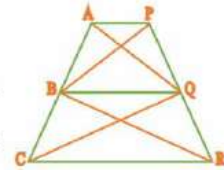
Triangles BQC and BQR are on the same base BQ and between same parallels $BQ \parallel CR$.

Hence, $ar(BQC) = ar(BQR)$... (2)

Adding equation (1) and (2), we get

$$ar(ABQ) + ar(BQC) = ar(PBQ) + ar(BQR)$$

$$\Rightarrow ar(AQC) = ar(PBR)$$

**Question 15:**

Diagonals AC and BD of a quadrilateral $ABCD$ intersect at O in such a way that $ar(AOD) = ar(BOC)$. Prove that $ABCD$ is a trapezium.

Answer 15:

$$ar(AOD) = ar(BOC) \quad [\because \text{Given}]$$

Adding $ar(AOB)$ both the sides

$$ar(AOD) + ar(AOB) = ar(BOC) + ar(AOB)$$

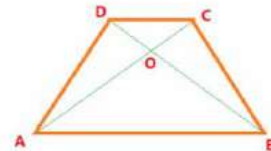
$$\Rightarrow ar(ABD) = ar(ABC)$$

ΔABD and ΔABC are on the same base AB and $ar(ABD) = ar(ABC)$.

Therefore, $AB \parallel DC$

[$\because$ Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]

Hence, $ABCD$ is a trapezium.

**Question 16:**

In Figure, $ar(DRC) = ar(DPC)$ and $ar(BDP) = ar(ARC)$. Show that both the quadrilaterals $ABCD$ and $DCPR$ are trapeziums.

Answer 16:

$$ar(DRC) = ar(DPC) \quad \dots (1) \quad [\because \text{Given}]$$

ΔDRC and ΔDPC are on the same base DC and $ar(DRC) = ar(DPC)$.

Therefore, $DC \parallel RP$

[$\because$ Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]

Hence, $DCPR$ is a trapezium.

$$\text{And } ar(ARC) = ar(BDP) \quad \dots (2) \quad [\because \text{Given}]$$

Subtracting equation (1) from equation (2), we get

$$ar(ARC) - ar(DRC) = ar(BDP) - ar(DPC)$$

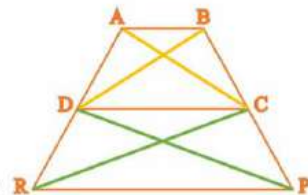
$$\Rightarrow ar(ADC) = ar(BDC)$$

ΔADC and ΔBDC are on the same base DC and $ar(ADC) = ar(BDC)$.

Therefore, $AB \parallel DC$

[$\because$ Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]

Hence, $ABCD$ is a trapezium.



Mathematics

(Chapter – 9)(Areas of Parallelograms and Triangles)

(Class – 9)

Exercise 9.4 (Optional)

Question 1:

Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Answer 1:

In $\triangle AFD$,

$$\angle F = 90^\circ$$

[$\because$ Angle of a rectangle]

$$AD > AF$$

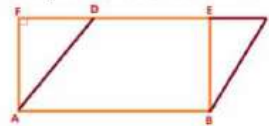
[$\because$ In a right triangle, hypotenuse is the longest side]

Adding AB on both the sides, $AD + AB > AF + AB$

Multiplying both sides by 2,

$$2[AD + AB] > 2[AF + AB]$$

$\Rightarrow$ Perimeter of parallelogram > Perimeter of rectangle

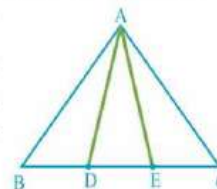


Question 2:

In Figure, D and E are two points on BC such that $BD = DE = EC$. Show that $\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$.

Can you now answer the question that you have left in the 'Introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?

[Remark: Note that by taking $BD = DE = EC$, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide $\triangle ABC$ into n triangles of equal areas.]



Answer 2:

In $\triangle ABC$, AD is median.

$$[\because BD = DE]$$

Hence, $\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE)$

$$\dots (1)$$

[$\because$ A median of a triangle divides it into two triangles of equal areas.]

Similarly, in $\triangle ADC$, AE is median.

$$[\because DE = EC]$$

Hence, $\text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$

$$\dots (2)$$

From the equation (1) and (2), $\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$

Question 3:

In Figure, ABCD, DCFE and ABFE are parallelograms. Show that $\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$.

Answer 3:

In $\triangle ADE$ and $\triangle BCF$,

$$AD = BC$$

[$\because$ Opposite sides of parallelogram ABCD]

$$DE = CF$$

[$\because$ Opposite sides of parallelogram DCFE]

$$AE = BF$$

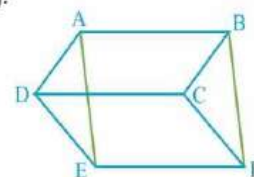
[$\because$ Opposite sides of parallelogram ABFE]

Hence, $\triangle ADE \cong \triangle BCF$

[$\because$ SSS Congruency rule]

Hence, $\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$

[$\because$ Congruent triangles are equal in area also]



Question 4:

In Figure, ABCD is a parallelogram and BC is produced to a point Q such that $AD = CQ$. If AQ intersect DC at P, show that $\text{ar}(\triangle BPC) = \text{ar}(\triangle DPQ)$.

[Hint : Join AC.]

Answer 4:

In $\triangle ADP$ and $\triangle QCP$,

$$\angle APD = \angle QPC$$

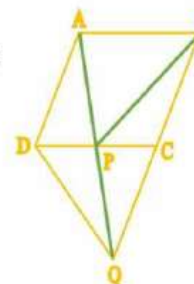
[$\because$ Vertically Opposite Angles]

$$\angle ADP = \angle QCP$$

[$\because$ Alternate angles]

$$AD = CQ$$

[$\because$ Given]



Hence, $\triangle ABD \cong \triangle ACD$ [$\because$ AAS Congruency rule]
 Therefore, $DP = CP$ [$\because$ CPCT]
 In $\triangle CDQ$, QP is median. [$\because DP = CP$]
 Hence, $ar(DPQ) = ar(QPC)$... (1)
 [$\because$ A median of a triangle divides it into two triangles of equal areas.]

Similarly,
 In $\triangle PBQ$, PC is median. [$\because AD = CQ$ and $AD = BC \Rightarrow BC = QC$]
 Hence, $ar(QPC) = ar(BPC)$... (2)
 From the equation (1) and (2),
 $ar(BPC) = ar(DPQ)$

Question 5:

In Figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC . If AE intersects BC at F , show that

- (i) $ar(BDE) = \frac{1}{4} ar(ABC)$ (ii) $ar(BDE) = \frac{1}{2} ar(BAE)$
 (iii) $ar(ABC) = 2 ar(BEC)$ (iv) $ar(BFE) = ar(AFD)$
 (v) $ar(BFE) = 2 ar(FED)$ (vi) $ar(FED) = \frac{1}{8} ar(AFC)$

[Hint: Join EC and AD . Show that $BE \parallel AC$ and $DE \parallel AB$, etc.]

Answer 5:

(i) **Construction:** Join EC and AD .

Let, $BC = x$

Therefore, $ar(ABC) = \frac{\sqrt{3}}{4} x^2$ [$\because$ Area of equilateral triangle $= \frac{\sqrt{3}}{4} (\text{side})^2$]

And $ar(BDE) = \frac{\sqrt{3}}{4} \left(\frac{x}{2}\right)^2$ [$\because D$ is mid-point of BC]

$= \frac{1}{4} \left[\frac{\sqrt{3}}{4} x^2 \right] = \frac{1}{4} [ar(ABC)]$

(ii) In $\triangle BEC$, ED is median. [$\because D$ is mid-point of BC]

Hence, $ar(BDE) = \frac{1}{2} ar(BEC)$... (1)

[$\because$ A median of a triangle divides it into two triangles of equal areas.]

$\angle EBC = 60^\circ$ and $\angle BCA = 60^\circ$ [$\because$ Angles of equilateral triangles]

Therefore, $\angle EBC = \angle BCA$

Here, Alternate angles ($\angle EBC = \angle BCA$) are equal, Hence, $BE \parallel AC$

Triangles BEC and BAE are on the same base BE and between same parallels, $BE \parallel AC$.

Hence, $ar(BEC) = ar(BAE)$... (2)

[$\because$ Triangles on the same base (or equal bases) and between the same parallels are equal in]

From the equation (1) and (2),

$ar(BDE) = \frac{1}{2} ar(BAE)$

(iii) In $\triangle BEC$, ED is median. [$\because D$ is mid-point of BC]

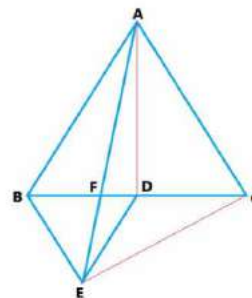
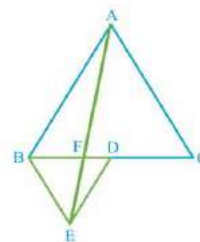
Hence, $ar(BDE) = \frac{1}{2} ar(BEC)$... (3)

[$\because$ A median of a triangle divides it into two triangles of equal areas.]

$ar(BDE) = \frac{1}{4} ar(ABC)$... (4) [$\because$ Proved above in (i)]

From the equation (3) and (4),

$ar(ABC) = 2 ar(BEC)$



(iv) $\angle ABD = 60^\circ$ and $\angle BDE = 60^\circ$ [$\because$ Angles of equilateral triangle]

Therefore, $\angle ABD = \angle BDE$

Here, Alternate angles ($\angle ABD = \angle BDE$) are equal,

Hence, $BA \parallel ED$

Triangles BDE and AED are on the same base ED and between same parallels $BA \parallel ED$.

Hence, $ar(BDE) = ar(AED)$

[$\because$ Triangles on the same base (or equal bases) and between the same parallels are equal in]

Subtracting $ar(FED)$ from both the sides

$$ar(BDE) - ar(FED) = ar(AED) - ar(FED)$$

$$\Rightarrow ar(BEF) = ar(AFD)$$

$$(v) \text{ In } \triangle BEC, AD^2 = AB^2 - BD^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4} \Rightarrow AD = \frac{\sqrt{3}a}{2}$$

$$\text{In } \triangle LED, EL^2 = DE^2 - DL^2 = \left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2 = \frac{a^2}{4} - \frac{a^2}{16} = \frac{3a^2}{16} \Rightarrow EL = \frac{\sqrt{3}a}{4}$$

$$\text{Therefore, } ar(AFD) = \frac{1}{2} \times FD \times AD = \frac{1}{2} \times FD \times \frac{\sqrt{3}a}{2} \quad \dots (5)$$

$$\text{And } ar(EFD) = \frac{1}{2} \times FD \times EL = \frac{1}{2} \times FD \times \frac{\sqrt{3}a}{4} \quad \dots (6)$$

From the equation (5) and (6),

$$ar(AFD) = 2 ar(FED)$$

$$\Rightarrow ar(BFE) = 2 ar(FED)$$

... (7)

[$\because$ Comparing with (iv)]

$$(vi) ar(BDE) = \frac{1}{4} ar(ABC)$$

[$\because$ From the equation (i)]

$$\Rightarrow ar(BEF) + ar(FED) = \frac{1}{4} ar(ABC)$$

$$\Rightarrow ar(BEF) + ar(FED) = \frac{1}{4} [2 ar(ADC)]$$

[$\because ar(ABC) = 2 ar(ADC)$]

$$\Rightarrow 2 ar(FED) + ar(FED) = \frac{1}{2} [ar(ADC)]$$

[$\because$ From the equation (v)]

$$\Rightarrow 3 ar(FED) = \frac{1}{2} [ar(AFC) - ar(AFD)]$$

$$\Rightarrow 3 ar(FED) = \frac{1}{2} [ar(AFC) - 2 ar(FED)]$$

[$\because$ From the equation (7)]

$$\Rightarrow 3 ar(FED) = \frac{1}{2} ar(AFC) - \frac{1}{2} \times 2 ar(FED)$$

$$\Rightarrow 3 ar(FED) = \frac{1}{2} ar(AFC) - ar(FED)$$

$$\Rightarrow 4 ar(FED) = \frac{1}{2} ar(AFC)$$

$$\Rightarrow ar(FED) = \frac{1}{8} ar(AFC)$$

Question 6:

Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that $ar(APB) \times ar(CPD) = ar(APD) \times ar(BPC)$. [Hint: From A and C, draw perpendiculars to BD.]

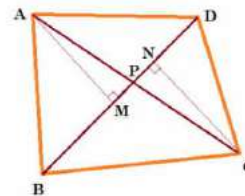
Answer 6:

Construction: From the points A and C, draw perpendiculars AM and CN on BD.

$$ar(APB) \times ar(CPD) = \frac{1}{2} \times BP \times AM \times \frac{1}{2} \times PD \times CN \quad \dots (1)$$

$$ar(APD) \times ar(BPC) = \frac{1}{2} \times PD \times AM \times \frac{1}{2} \times BP \times CN \quad \dots (2)$$

From the equation (1) and (2), $ar(APB) \times ar(CPD) = ar(APD) \times ar(BPC)$



Question 7:

P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

$$(i) \ar(PQR) = \frac{1}{2} \ar(ARC)$$

$$(ii) \ar(RQC) = \frac{3}{8} \ar(ABC)$$

$$(iii) \ar(PBQ) = \ar(ARC)$$

Answer 7:

Construction: Join AQ, PC, RC and RQ.

(i) In $\triangle APQ$, QR is median. [$\because$ Given]

$$\text{Hence, } \ar(PQR) = \frac{1}{2} \ar(APQ) \quad \dots (1)$$

[$\because$ A median of a triangle divides it into two triangles of equal areas.]

Similarly,

In $\triangle AQB$, QP is median. [$\because$ Given]

$$\text{Hence, } \ar(APQ) = \frac{1}{2} \ar(ABQ) \quad \dots (2)$$

And, in $\triangle ABC$, AQ is median. [$\because$ Given]

$$\text{Hence, } \ar(ABQ) = \frac{1}{2} \ar(ABC) \quad \dots (3)$$

From the equation (1), (2) and (3),

$$\ar(PQR) = \frac{1}{8} \ar(ABC) \quad \dots (4)$$

In $\triangle ARC$, CR is median. [$\because$ Given]

$$\text{Hence, } \ar(ARC) = \frac{1}{2} \ar(APC) \quad \dots (5)$$

[$\because$ A median of a triangle divides it into two triangles of equal areas.]

Similarly,

In $\triangle ABC$, CP is median. [$\because$ Given]

$$\text{Hence, } \ar(APC) = \frac{1}{2} \ar(ABC) \quad \dots (6)$$

From the equation (5) and (6),

$$\ar(ARC) = \frac{1}{4} \ar(ABC) \quad \dots (7)$$

From the equation (4) and (7),

$$\ar(PQR) = \frac{1}{8} \ar(ABC) = \frac{1}{2} \left[\frac{1}{4} \ar(ABC) \right] = \frac{1}{2} \ar(ARC)$$

$$(ii) \ar(RQC) = \ar(RQA) + \ar(AQC) - \ar(ARC) \quad \dots (8)$$

In $\triangle PQA$, QR is median. [$\because$ Given]

$$\text{Hence, } \ar(RQA) = \frac{1}{2} \ar(PQA) \quad \dots (9)$$

In $\triangle AQB$, PQ is median.

$$\text{Hence, } \ar(PQA) = \frac{1}{2} \ar(AQB) \quad \dots (10)$$

In $\triangle ABC$, AQ is median. [$\because$ Given]

$$\text{Hence, } \ar(AQB) = \frac{1}{2} \ar(ABC) \quad \dots (11)$$

From the equation (9), (10) and (11),

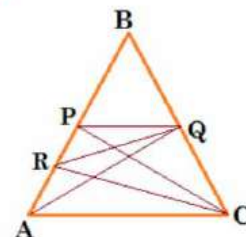
$$\ar(RQA) = \frac{1}{8} \ar(ABC) \quad \dots (12)$$

In $\triangle ABC$, AQ is median. [$\because$ Given]

$$\text{Hence, } \ar(AQC) = \frac{1}{2} \ar(ABC) \quad \dots (13)$$

In $\triangle APC$, CR is median.

$$\text{Hence, } \ar(ARC) = \frac{1}{2} \ar(APC) \quad \dots (14)$$



In $\triangle ABC$, CP is median. [$\because$ Given]

Hence, $ar(APC) = \frac{1}{2} ar(ABC)$... (15)

From the equation (14) and (15),

$ar(ARC) = \frac{1}{4} ar(ABC)$... (16)

From the equation (8), (12), (13) and (16),

$$ar(RQC) = \frac{1}{8} ar(ABC) + \frac{1}{2} ar(ABC) - \frac{1}{4} ar(ABC) = \frac{3}{8} ar(ABC)$$

(iii) In $\triangle ABQ$, PQ is median. [$\because$ Given]

Hence, $ar(PBQ) = \frac{1}{2} ar(ABQ)$... (17)

In $\triangle ABC$, AQ is median.

Hence, $ar(ABQ) = \frac{1}{2} ar(ABC)$... (18)

From the equation (16), (17) and (18),

$ar(PQB) = ar(ARC)$

Question 8:

In Figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment $AX \perp DE$ meets BC at Y. Show that:

(i) $\triangle MBC \cong \triangle ABD$

(ii) $ar(BYXD) = 2 ar(MBC)$

(iii) $ar(BYXD) = ar(ABMN)$

(iv) $\triangle FCB \cong \triangle ACE$

(v) $ar(CYXE) = 2 ar(FCB)$

(vi) $ar(CYXE) = ar(ACFG)$

(vii) $ar(BCED) = ar(ABMN) + ar(ACFG)$

Answer 8:

(i) In $\triangle MBC$ and $\triangle ABD$,

$AB = AC$ [$\because$ Sides of square]

$\angle MBC = \angle ABD$ [$\because$ Each $90^\circ + \angle ABC$]

$MB = AB$ [$\because$ Sides of square]

Hence, $\triangle MBC \cong \triangle ABD$ [$\because$ SAS Congruency rule]

(ii) Triangle ABD and parallelogram BYXD are on the same base BD and between same parallels $AX \parallel BD$.

Hence, $ar(ABD) = \frac{1}{2} ar(BYXD)$... (1)

[$\because$ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

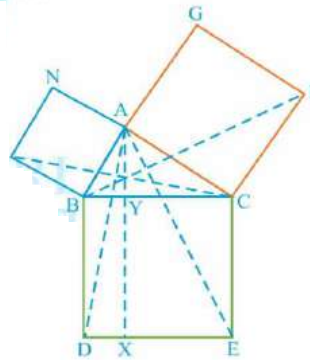
But, $\triangle MBC \cong \triangle ABD$ [$\because$ Proved above]

Therefore, $ar(MBC) = ar(ABD)$... (2)

From the equation (1) and (2),

$ar(MBC) = \frac{1}{2} ar(BYXD)$... (3)

$\Rightarrow 2 ar(MBC) = ar(BYXD)$



(iii) Triangle MBC and square ABMN are on the same base MB and between same parallels MB || NC.

$$\text{Hence, } ar(MBC) = \frac{1}{2} ar(ABMN) \quad \dots (4)$$

[$\because$ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

From the equation (3) and (4),

$$ar(BYXD) = ar(ABMN)$$

(iv) In $\triangle ACE$ and $\triangle BCF$,

$$CE = BC \quad [\because \text{Sides of square}]$$

$$\angle ACE = \angle BCF \quad [\because \text{Each } 90^\circ + \angle BCA]$$

$$AC = CF \quad [\because \text{Sides of square}]$$

$$\text{Hence, } \triangle ACE \cong \triangle BCF \quad [\because \text{SAS Congruency rule}]$$

(v) Triangle ACE and square CYXE are on the same base CE and between same parallels CE || AX.

$$\text{Hence, } ar(ACE) = \frac{1}{2} ar(CYXE)$$

[$\because$ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

$$\Rightarrow ar(FCB) = \frac{1}{2} ar(CYXE) \quad \dots (5) \quad [\because ar(FCB) = ar(ACE)]$$

$$\Rightarrow 2 ar(FCB) = ar(CYXE)$$

(vi) Triangle BCF and square ACFG are on the same base CF and between same parallels CF || FG.

$$\text{Hence, } ar(BCF) = \frac{1}{2} ar(ACFG) \quad \dots (6)$$

[$\because$ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

From the equation (5) and (6),

$$\Rightarrow ar(CYXE) = ar(ACFG)$$

(vii) From the result of **(iii)**, we have

$$ar(BYXD) = ar(ABMN) \quad \dots (7)$$

From the result of **(vi)**, we have

$$ar(CYXE) = ar(ACFG) \quad \dots (8)$$

Adding (7) and (8), we get

$$ar(BYXD) + ar(CYXE) = ar(ABMN) + ar(ACFG)$$

$$\Rightarrow ar(BCED) = ar(ABMN) + ar(ACFG)$$