

Mathematics

(Chapter – 6) (The Triangle and its Properties) (Class – VII)

Exercise 6.1

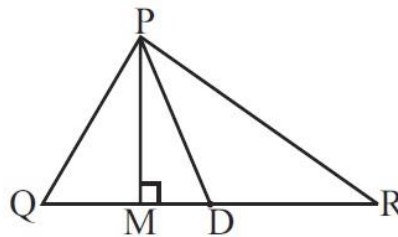
Question 1:

In $\triangle PQR$, D is the mid-point of $\overline{QR}$.

$\overline{PM}$ is _____

PD is _____

Is $QM = MR$?



Answer 1:

Given: $QD = DR$

$\therefore \overline{PM}$ is altitude.

PD is median.

No, $QM \neq MR$ as D is the mid-point of QR.

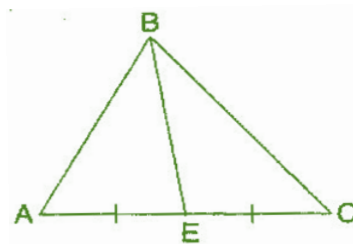
Question 2:

Draw rough sketches for the following:

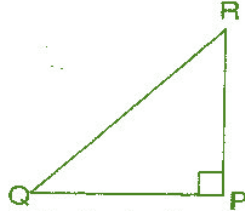
- (a) In $\triangle ABC$, BE is a median.
- (b) In $\triangle PQR$, PQ and PR are altitudes of the triangle.
- (c) In $\triangle XYZ$, YL is an altitude in the exterior of the triangle.

Answer 2:

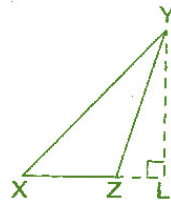
(a) Here, BE is a median in $\triangle ABC$ and $AE = EC$.



(b) Here, PQ and PR are the altitudes of the $\triangle PQR$ and $RP \perp QP$.



(c) YL is an altitude in the exterior of $\triangle XYZ$.



Question 3:

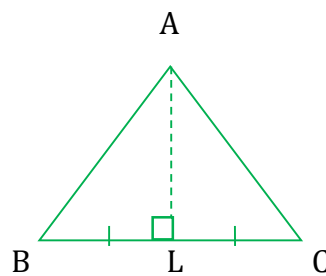
Verify by drawing a diagram if the median and altitude of an isosceles triangle can be the same.

Answer 3:

Isosceles triangle means any two sides are equal.

Take $\triangle ABC$ and draw the median when $AB = AC$.

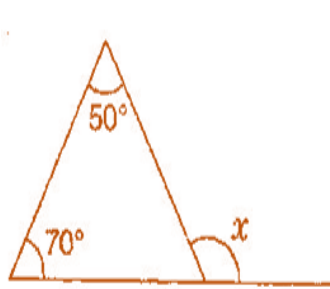
AL is the median and altitude of the given triangle.



Exercise 6.2

Question 1:

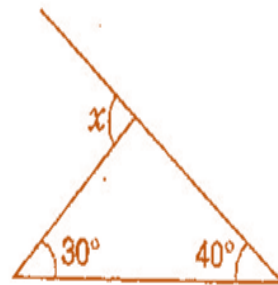
Find the value of the unknown exterior angle x in the following diagrams:



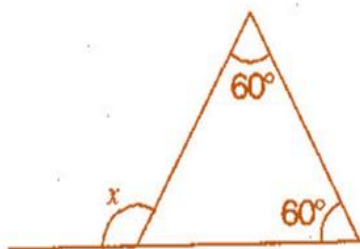
(i)



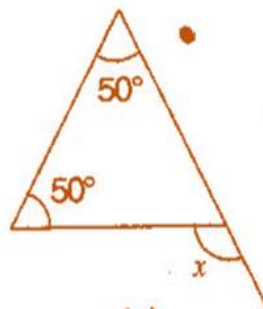
(ii)



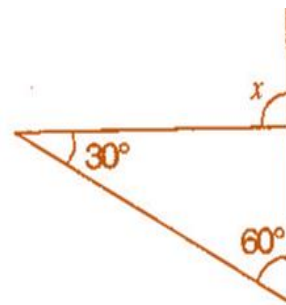
(iii)



(iv)



(v)



(vi)

Answer 1:

Since, Exterior angle = Sum of interior opposite angles, therefore

(i) $x = 50^\circ + 70^\circ = 120^\circ$

(ii) $x = 65^\circ + 45^\circ = 110^\circ$

(iii) $x = 30^\circ + 40^\circ = 70^\circ$

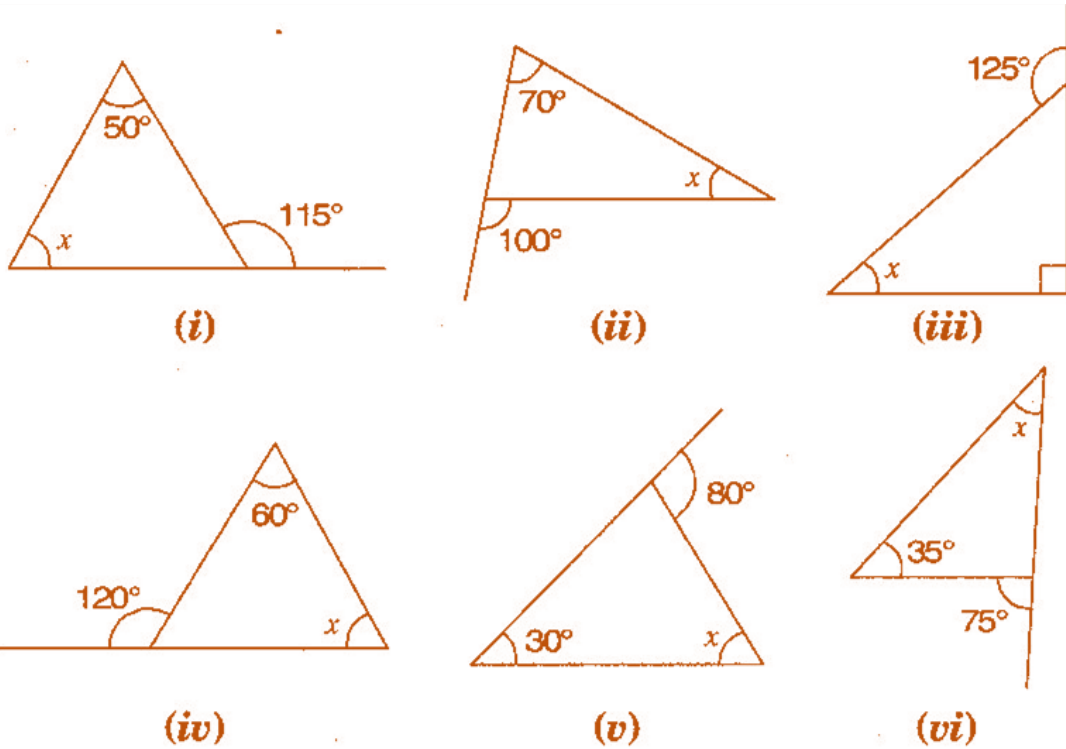
(iv) $x = 60^\circ + 60^\circ = 120^\circ$

(v) $x = 50^\circ + 50^\circ = 100^\circ$

(vi) $x = 60^\circ + 30^\circ = 90^\circ$

Question 2:

Find the value of the unknown interior angle x in the following figures:

**Answer 2:**

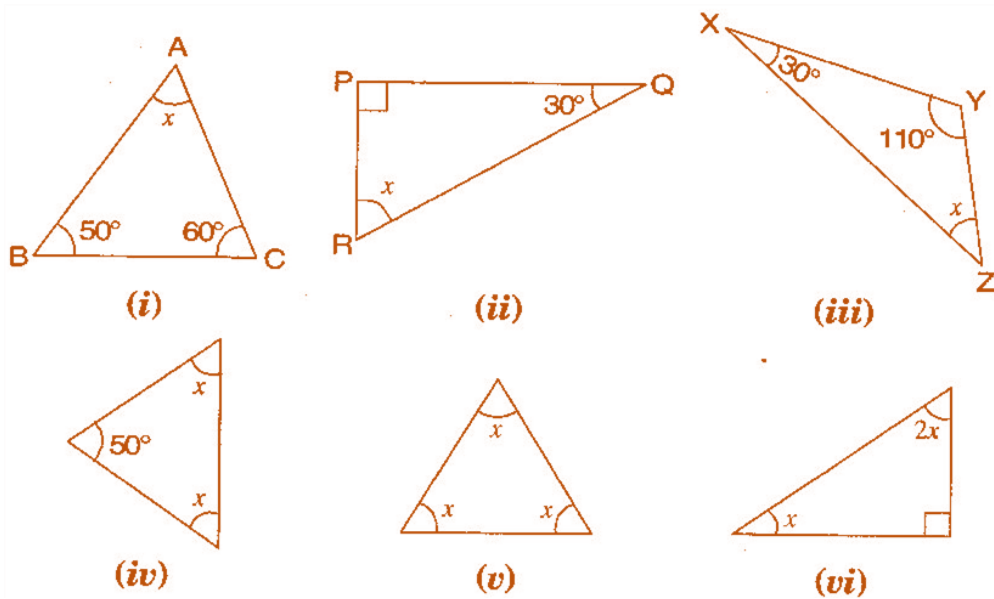
Since, Exterior angle = Sum of interior opposite angles, therefore

- | | | | |
|-------|----------------------------|---------------|---------------------------------------|
| (i) | $x + 50^\circ = 115^\circ$ | $\Rightarrow$ | $x = 115^\circ - 50^\circ = 65^\circ$ |
| (ii) | $70^\circ + x = 100^\circ$ | $\Rightarrow$ | $x = 100^\circ - 70^\circ = 30^\circ$ |
| (iii) | $x + 90^\circ = 125^\circ$ | $\Rightarrow$ | $x = 125^\circ - 90^\circ = 35^\circ$ |
| (iv) | $60^\circ + x = 120^\circ$ | $\Rightarrow$ | $x = 120^\circ - 60^\circ = 60^\circ$ |
| (v) | $30^\circ + x = 80^\circ$ | $\Rightarrow$ | $x = 80^\circ - 30^\circ = 50^\circ$ |
| (vi) | $x + 35^\circ = 75^\circ$ | $\Rightarrow$ | $x = 75^\circ - 35^\circ = 40^\circ$ |

Exercise 6.3

Question 1:

Find the value of unknown x in the following diagrams:



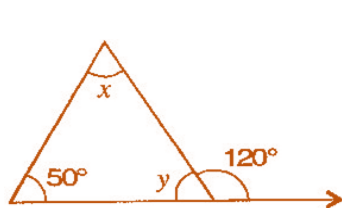
Answer 1:

- (i) In $\triangle ABC$,
 $\angle BAC + \angle ACB + \angle ABC = 180^\circ$ [By angle sum property of a triangle]
 $\Rightarrow x + 50^\circ + 60^\circ = 180^\circ$
 $\Rightarrow x + 110^\circ = 180^\circ$
 $\Rightarrow x = 180^\circ - 110^\circ = 70^\circ$
- (ii) In $\triangle PQR$,
 $\angle RPQ + \angle PQR + \angle RPQ = 180^\circ$ [By angle sum property of a triangle]
 $\Rightarrow 90^\circ + 30^\circ + x = 180^\circ$
 $\Rightarrow x + 120^\circ = 180^\circ$
 $\Rightarrow x = 180^\circ - 120^\circ = 60^\circ$
- (iii) In $\triangle XYZ$,
 $\angle ZXY + \angle XYZ + \angle YZX = 180^\circ$ [By angle sum property of a triangle]
 $\Rightarrow 30^\circ + 110^\circ + x = 180^\circ$
 $\Rightarrow x + 140^\circ = 180^\circ$
 $\Rightarrow x = 180^\circ - 140^\circ = 40^\circ$

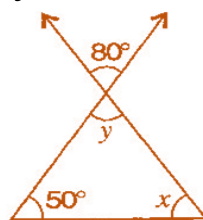
- (iv) In the given isosceles triangle,
 $x + x + 50^\circ = 180^\circ$ [By angle sum property of a triangle]
 $\Rightarrow 2x + 50^\circ = 180^\circ$
 $\Rightarrow 2x = 180^\circ - 50^\circ$
 $\Rightarrow 2x = 130^\circ$
 $\Rightarrow x = \frac{130^\circ}{2} = 65^\circ$
- (v) In the given equilateral triangle,
 $x + x + x = 180^\circ$ [By angle sum property of a triangle]
 $\Rightarrow 3x = 180^\circ$
 $\Rightarrow x = \frac{180^\circ}{3} = 60^\circ$
- (vi) In the given right angled triangle,
 $x + 2x + 90^\circ = 180^\circ$ [By angle sum property of a triangle]
 $\Rightarrow 3x + 90^\circ = 180^\circ$
 $\Rightarrow 3x = 180^\circ - 90^\circ$
 $\Rightarrow 3x = 90^\circ$
 $\Rightarrow x = \frac{90^\circ}{3} = 30^\circ$

Question 2:

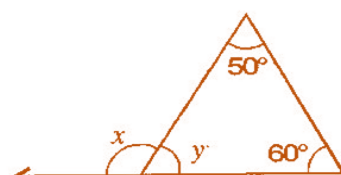
Find the values of the unknowns x and y in the following diagrams:



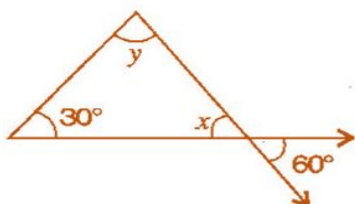
(i)



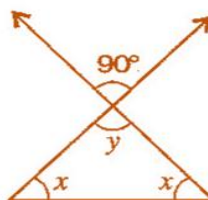
(ii)



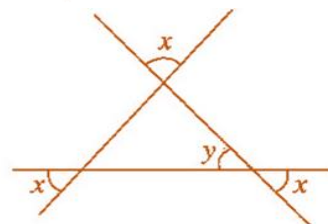
(iii)



(iv)



(v)



(vi)

Answer 2:

- (i) $50^\circ + x = 120^\circ$ [Exterior angle property of a Δ]
 $\Rightarrow x = 120^\circ - 50^\circ = 70^\circ$
Now, $50^\circ + x + y = 180^\circ$ [Angle sum property of a Δ]
 $\Rightarrow 50^\circ + 70^\circ + y = 180^\circ$
 $\Rightarrow 120^\circ + y = 180^\circ$
 $\Rightarrow y = 180^\circ - 120^\circ = 60^\circ$
- (ii) $y = 80^\circ$ (i) [Vertically opposite angle]
Now, $50^\circ + x + y = 180^\circ$ [Angle sum property of a Δ]
 $\Rightarrow 50^\circ + 80^\circ + y = 180^\circ$ [From equation (i)]
 $\Rightarrow 130^\circ + y = 180^\circ$
 $\Rightarrow y = 180^\circ - 130^\circ = 50^\circ$
- (iii) $50^\circ + 60^\circ = x$ [Exterior angle property of a Δ]
 $\Rightarrow x = 110^\circ$
Now $50^\circ + 60^\circ + y = 180^\circ$ [Angle sum property of a Δ]
 $\Rightarrow 110^\circ + y = 180^\circ$
 $\Rightarrow y = 180^\circ - 110^\circ$
 $\Rightarrow y = 70^\circ$
- (iv) $x = 60^\circ$ (i) [Vertically opposite angle]
Now, $30^\circ + x + y = 180^\circ$ [Angle sum property of a Δ]
 $\Rightarrow 50^\circ + 60^\circ + y = 180^\circ$ [From equation (i)]
 $\Rightarrow 90^\circ + y = 180^\circ$
 $\Rightarrow y = 180^\circ - 90^\circ = 90^\circ$
- (v) $y = 90^\circ$ (i) [Vertically opposite angle]
Now, $y + x + x = 180^\circ$ [Angle sum property of a Δ]
 $\Rightarrow 90^\circ + 2x = 180^\circ$ [From equation (i)]
 $\Rightarrow 2x = 180^\circ - 90^\circ$
 $\Rightarrow 2x = 90^\circ$
 $\Rightarrow x = \frac{90^\circ}{2} = 45^\circ$

(vi) $x = y$ (i)

[Vertically opposite angle]

Now, $x + x + y = 180^\circ$

[Angle sum property of a Δ]

$\Rightarrow 2x + x = 180^\circ$

[From equation (i)]

$\Rightarrow 3x = 180^\circ$

$\Rightarrow x = \frac{180^\circ}{3} = 60^\circ$

Exercise 6.4

Question 1:

Is it possible to have a triangle with the following sides?

- (i) 2 cm, 3 cm, 5 cm
- (ii) 3 cm, 6 cm, 7 cm
- (iii) 6 cm, 3 cm, 2 cm

Answer 1:

Since, a triangle is possible whose sum of the lengths of any two sides would be greater than the length of third side.

- (i) 2 cm, 3 cm, 5 cm

$$2 + 3 > 5 \quad \text{No}$$

$$2 + 5 > 3 \quad \text{Yes}$$

$$3 + 5 > 2 \quad \text{Yes}$$

This triangle is not possible.

- (ii) 3 cm, 6 cm, 7 cm

$$3 + 6 > 7 \quad \text{Yes}$$

$$6 + 7 > 3 \quad \text{Yes}$$

$$3 + 7 > 6 \quad \text{Yes}$$

This triangle is possible.

- (iii) 6 cm, 3 cm, 2 cm

$$6 + 3 > 2 \quad \text{Yes}$$

$$6 + 2 > 3 \quad \text{Yes}$$

$$2 + 3 > 6 \quad \text{No}$$

This triangle is not possible.

Question 2:

Take any point O in the interior of a triangle PQR. Is:

- (i) $OP + OQ > PQ$?

- (ii) $OQ + OR > QR$?

- (iii) $OR + OP > RP$?

Answer 2:

Join OR, OQ and OP.

- (i) Is $OP + OQ > PQ$?

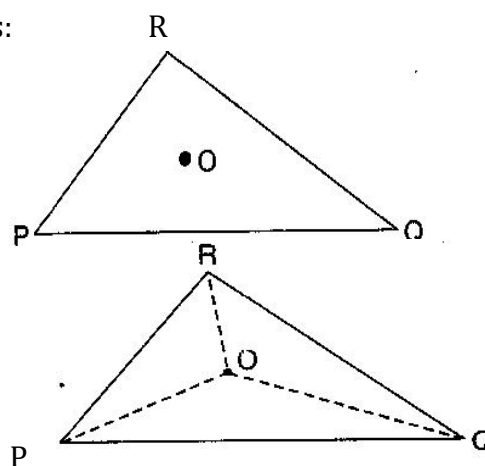
Yes, POQ form a triangle.

- (ii) Is $OQ + OR > QR$?

Yes, RQO form a triangle.

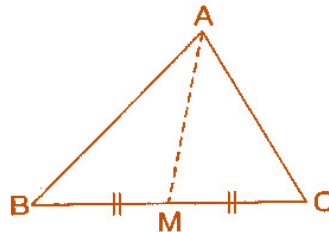
- (iii) Is $OR + OP > RP$?

Yes, ROP form a triangle.



Question 3:

AM is a median of a triangle ABC. Is $AB + BC + CA > 2AM$? (Consider the sides of triangles $\triangle ABM$ and $\triangle AMC$.)

**Answer 3:**

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.

$$\begin{array}{lll} \text{Therefore,} & \text{In } \triangle ABM, & AB + BM > AM & \dots (i) \\ & \text{In } \triangle AMC, & AC + MC > AM & \dots (ii) \end{array}$$

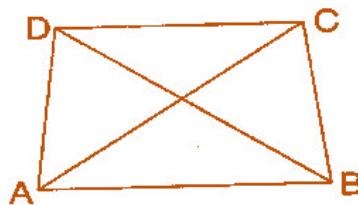
Adding eq. (i) and (ii),

$$\begin{aligned} & AB + BM + AC + MC > AM + AM \\ \Rightarrow & AB + AC + (BM + MC) > 2AM \\ \Rightarrow & AB + AC + BC > 2AM \end{aligned}$$

Hence, it is true.

Question 4:

ABCD is a quadrilateral. Is $AB + BC + CD + DA > AC + BD$?

**Answer 4:**

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.

$$\begin{array}{lll} \text{Therefore, In } \triangle ABC, & AB + BC > AC & \dots\dots\dots(i) \\ & \text{In } \triangle ADC, & AD + DC > AC & \dots\dots\dots(ii) \\ & \text{In } \triangle DCB, & DC + CB > DB & \dots\dots\dots(iii) \\ & \text{In } \triangle ADB, & AD + AB > DB & \dots\dots\dots(iv) \end{array}$$

Adding equations (i), (ii), (iii) and (iv), we get

$$\begin{aligned}
& AB + BC + AD + DC + DC + CB + AD + AB > AC + AC + DB + DB \\
\Rightarrow & (AB + AB) + (BC + BC) + (AD + AD) + (DC + DC) > 2AC + 2DB \\
\Rightarrow & 2AB + 2BC + 2AD + 2DC > 2(AC + DB) \\
\Rightarrow & 2(AB + BC + AD + DC) > 2(AC + DB) \\
\Rightarrow & AB + BC + AD + DC > AC + DB \\
\Rightarrow & AB + BC + CD + DA > AC + DB
\end{aligned}$$

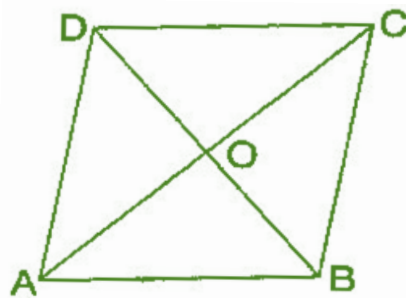
Hence, it is true.

Question 5:

ABCD is quadrilateral. Is $AB + BC + CD + DA < 2(AC + BD)$?

Answer 5:

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.



$$\begin{aligned}
\text{Therefore, In } \triangle AOB, \quad AB &< OA + OB && \text{.....(i)} \\
\text{In } \triangle BOC, \quad BC &< OB + OC && \text{.....(ii)} \\
\text{In } \triangle COD, \quad CD &< OC + OD && \text{.....(iii)} \\
\text{In } \triangle AOD, \quad DA &< OD + OA && \text{.....(iv)} \\
\text{Adding equations (i), (ii), (iii) and (iv), we get} \\
AB + BC + CD + DA &< OA + OB + OB + OC + OC + OD + OD + OA \\
\Rightarrow AB + BC + CD + DA &< 2OA + 2OB + 2OC + 2OD \\
\Rightarrow AB + BC + CD + DA &< 2[(AO + OC) + (DO + OB)] \\
\Rightarrow AB + BC + CD + DA &< 2(AC + BD)
\end{aligned}$$

Hence, it is proved.

Question 6:

The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Answer 6:

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.

It is given that two sides of triangle are 12 cm and 15 cm.

Therefore, the third side should be less than $12 + 15 = 27$ cm.

And also the third side cannot be less than the difference of the two sides.

Therefore, the third side has to be more than $15 - 12 = 3$ cm.

Hence, the third side could be the length more than 3 cm and less than 27 cm.

Exercise 6.5

Question 1:

PQR is a triangle, right angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

Answer 1:

Given: PQ = 10 cm, PR = 24 cm

Let QR be x cm.

In right angled triangle QPR,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$\Rightarrow (QR)^2 = (PQ)^2 + (PR)^2$$

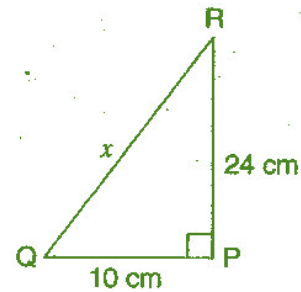
$$\Rightarrow x^2 = (10)^2 + (24)^2$$

$$\Rightarrow x^2 = 100 + 576 = 676$$

$$\Rightarrow x = \sqrt{676} = 26 \text{ cm}$$

Thus, the length of QR is 26 cm.

[By Pythagoras theorem]



Question 2:

ABC is a triangle, right angled at C. If AB = 25 cm and AC = 7 cm, find BC.

Answer 2:

Given: AB = 25 cm, AC = 7 cm

Let BC be x cm.

In right angled triangle ACB,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$\Rightarrow (AB)^2 = (AC)^2 + (BC)^2$$

$$\Rightarrow (25)^2 = (7)^2 + x^2$$

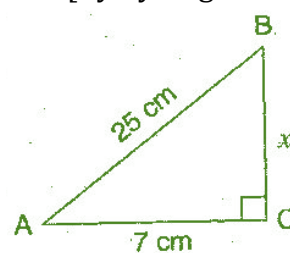
$$\Rightarrow 625 = 49 + x^2$$

$$\Rightarrow x^2 = 625 - 49 = 576$$

$$\Rightarrow x = \sqrt{576} = 24 \text{ cm}$$

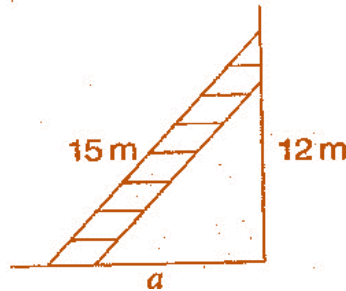
Thus, the length of BC is 24 cm.

[By Pythagoras theorem]



Question 3:

A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a . Find the distance of the foot of the ladder from the wall.

**Answer 3:**

Let AC be the ladder and A be the window.

Given: AC = 15 m, AB = 12 m, CB = a m

In right angled triangle ACB,

(Hypotenuse)² = (Base)² + (Perpendicular)²

[By Pythagoras theorem]

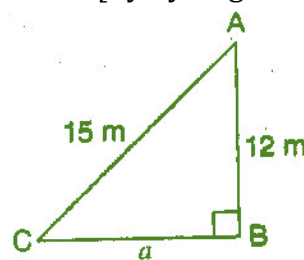
$$\Rightarrow (AC)^2 = (CB)^2 + (AB)^2$$

$$\Rightarrow (15)^2 = (a)^2 + (12)^2$$

$$\Rightarrow 225 = a^2 + 144$$

$$\Rightarrow a^2 = 225 - 144 = 81$$

$$\Rightarrow a = \sqrt{81} = 9 \text{ cm}$$



Thus, the distance of the foot of the ladder from the wall is 9 m.

Question 4:

Which of the following can be the sides of a right triangle?

(i) 2.5 cm, 6.5 cm, 6 cm

(ii) 2 cm, 2 cm, 5 cm

(iii) 1.5 cm, 2 cm, 2.5 cm

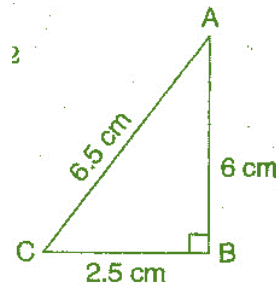
In the case of right angled triangles, identify the right angles.

Answer 4:

Let us consider, the larger side be the hypotenuse and also using Pythagoras theorem,

(Hypotenuse)² = (Base)² + (Perpendicular)²

- (i) 2.5 cm, 6.5 cm, 6 cm



$$\text{In } \triangle ABC, \quad (AC)^2 = (AB)^2 + (BC)^2$$

$$\text{L.H.S.} = (6.5)^2 = 42.25 \text{ cm}$$

$$\text{R.H.S.} = (6)^2 + (2.5)^2 = 36 + 6.25 = 42.25 \text{ cm}$$

Since, L.H.S. = R.H.S.

Therefore, the given sides are of the right angled triangle.

Right angle lies on the opposite to the greater side 6.5 cm, i.e., at B.

- (ii) 2 cm, 2 cm, 5 cm

$$\text{In the given triangle, } (5)^2 = (2)^2 + (2)^2$$

$$\text{L.H.S.} = (5)^2 = 25$$

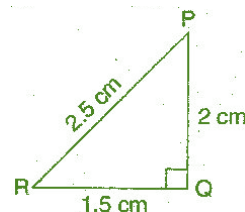
$$\text{R.H.S.} = (2)^2 + (2)^2 = 4 + 4 = 8$$

Since, L.H.S. $\neq$ R.H.S.

Therefore, the given sides are not of the right angled triangle.

- (iii) 1.5 cm, 2 cm, 2.5 cm

$$\text{In } \triangle PQR, \quad (PR)^2 = (PQ)^2 + (RQ)^2$$



$$\text{L.H.S.} = (2.5)^2 = 6.25 \text{ cm}$$

$$\text{R.H.S.} = (1.5)^2 + (2)^2 = 2.25 + 4 = 6.25 \text{ cm}$$

Since, L.H.S. = R.H.S.

Therefore, the given sides are of the right angled triangle.

Right angle lies on the opposite to the greater side 2.5 cm, i.e., at Q.

Question 5:

A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

Answer 5:

Let A'CB represents the tree before it broken at the point C and let the top A' touches the ground at A after it broke. Then $\triangle ABC$ is a right angled triangle, right angled at B.

AB = 12 m and BC = 5 m

Using Pythagoras theorem, In $\triangle ABC$

$$(AC)^2 = (AB)^2 + (BC)^2$$

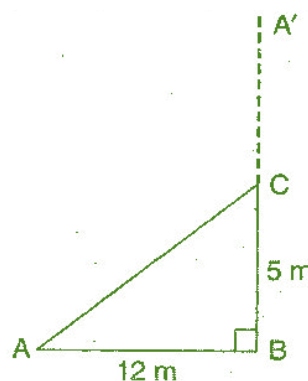
$$\Rightarrow (AC)^2 = (12)^2 + (5)^2$$

$$\Rightarrow (AC)^2 = 144 + 25$$

$$\Rightarrow (AC)^2 = 169$$

$$\Rightarrow AC = 13 \text{ m}$$

Hence, the total height of the tree = AC + CB = 13 + 5 = 18 m.

**Question 6:**

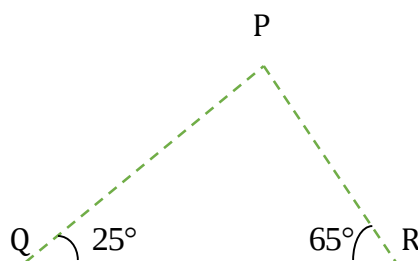
Angles Q and R of a $\triangle PQR$ are 25° and 65° .

Write which of the following is true:

(i) $PQ^2 + QR^2 = RP^2$

(ii) $PQ^2 + RP^2 = QR^2$

(iii) $RP^2 + QR^2 = PQ^2$

**Answer 6:**

In $\triangle PQR$,

$$\angle PQR + \angle QRP + \angle RPQ = 180^\circ$$

[By Angle sum property of a $\triangle$]

$$\Rightarrow 25^\circ + 65^\circ + \angle RPQ = 180^\circ$$

$$\Rightarrow 90^\circ + \angle RPQ = 180^\circ$$

$$\Rightarrow \angle RPQ = 180^\circ - 90^\circ = 90^\circ$$

Thus, $\triangle PQR$ is a right angled triangle, right angled at P.

$$\therefore (\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2 \quad [\text{By Pythagoras theorem}]$$

$$\Rightarrow (QR)^2 = (PR)^2 + (QP)^2$$

Hence, Option (ii) is correct.

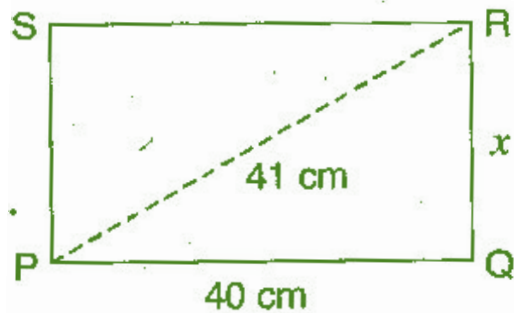
Question 7:

Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.

Answer 7:

Given diagonal (PR) = 41 cm, length (PQ) = 40 cm

Let breadth (QR) be x cm.



Now, in right angled triangle PQR,

$$(PR)^2 = (RQ)^2 + (PQ)^2 \quad \text{[By Pythagoras theorem]}$$

$$\Rightarrow (41)^2 = x^2 + (40)^2$$

$$\Rightarrow 1681 = x^2 + 1600$$

$$\Rightarrow x^2 = 1681 - 1600$$

$$\Rightarrow x^2 = 81$$

$$\Rightarrow x = \sqrt{81} = 9 \text{ cm}$$

Therefore the breadth of the rectangle is 9 cm.

$$\begin{aligned} \text{Perimeter of rectangle} &= 2(\text{length} + \text{breadth}) \\ &= 2(9 + 40) \\ &= 2 \times 49 = 98 \text{ cm} \end{aligned}$$

Hence, the perimeter of the rectangle is 98 cm.

Question 8:

The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

Answer 8:

Given: Diagonals AC = 30 cm and DB = 16 cm.

Since the diagonals of the rhombus bisect at right angle to each other.

Therefore, $OD = \frac{DB}{2} = \frac{16}{2} = 8 \text{ cm}$

And $OC = \frac{AC}{2} = \frac{30}{2} = 15 \text{ cm}$

Now, In right angle triangle DOC,

$$(DC)^2 = (OD)^2 + (OC)^2$$

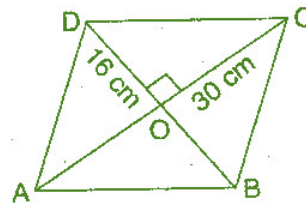
$$\Rightarrow (DC)^2 = (8)^2 + (15)^2$$

$$\Rightarrow (DC)^2 = 64 + 225 = 289$$

$$\Rightarrow DC = \sqrt{289} = 17 \text{ cm}$$

Perimeter of rhombus = $4 \times \text{side} = 4 \times 17 = 68 \text{ cm}$

Thus, the perimeter of rhombus is 68 cm.



[By Pythagoras theorem]