COORDINATE GEOMETRY MASTER SHEET

1. Distance Formula

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

(Distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$)

Key Point:
- Gives the shortest distance between two points.

Hint:
- Remember the pattern $(x_2 - x_1)^2 + (y_2 - y_1)^2$

2. Midpoint Formula

$$\boxed{M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

(Midpoint of line segment joining P and Q)

Key Point:
- Divides the line segment into two equal parts.

Hint:
- Add the x-coordinates and divide by 2, same for y-coordinates.

3. Section Formula (Internal Division)

$$\boxed{P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)}$$

($m:n \Rightarrow$ ratio in which P divides the line segment internally)

Key Point:
- Finds the coordinates of a point which divides the line internally.

Hint:
- Multiply the opposite coordinates by their ratio parts.

4. Slope of a Line

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}} \qquad (x_2 \neq x_1)$$

Key Point:
- Measures the steepness and direction of a line.

Hint:
- Rise $(y_2 - y_1)$ over Run $(x_2 - x_1)$

5. Equations of a Line

① Slope-intercept form : $\boxed{y = mx + c}$ (m = slope, c = y-intercept)

② Point-slope form : $\boxed{y - y_1 = m(x - x_1)}$ (line with slope m through (x_1, y_1))

③ Two-point form : $\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$ (line through (x_1, y_1) and (x_2, y_2))

Key Point:
- Different forms, same line! Use as per need.

Hint:
- Use slope-intercept for graphing. Use point-slope for one point and slope. Use two-point form for two given points.

6. Angle Between Two Lines

$$\boxed{\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|}, \quad 0 \leq \theta < 180^\circ$$

(m_1 and m_2 are slopes of the two lines)

Key Point:
- Gives the angle between two lines.

Hint:
- Use absolute value for positive angle.

7. Distance of a Point From a Line

$$\boxed{d = \left|\frac{Ax_2 + By_2 + C}{\sqrt{A^2 + B^2}}\right|}$$

(Distance of point $P(x_1, y_1)$ from the line $Ax + By + C = 0$)

Key Point:
- Perpendicular distance from a point to a line.

Hint:
- Always take absolute value.

These are the most important formulas of Coordinate Geometry.

Solve the following simultaneous linear equations.

Topic Box: SIMULTANEOUS LINEAR EQUATION

(a) $\begin{cases} 2x + 3y = 13 \\ x - y = 2 \end{cases}$

Solution:

From equation (ii): $x - y = 2 \Rightarrow x = y + 2$ ---- (i)

Substitute (i) into (ii):

$2(y+2) + 3y = 13$

$2y + 4 + 3y = 13$

$5y + 4 = 13$

$5y = 13 - 4 = 9$

$y = \frac{9}{5}$

Substitute $y = \frac{9}{5}$ into (i): $x = \frac{9}{5} + 2 = \frac{9}{5} + \frac{10}{5} = \frac{19}{5}$

$\therefore \quad x = \frac{19}{5}, \quad y = \frac{9}{5}$

(b) $\begin{cases} 3x + 2y = 16 \\ 2x - y = 1 \end{cases}$

Solution:

From equation (ii): $2x - y = 1 \Rightarrow y = 2x - 1$ ---- (i)

Substitute (i) into (ii):

$3x + 2(2x-1) = 16$

$3x + 4x - 2 = 16$

$7x - 2 = 16$

$7x = 18$

$x = \frac{18}{7}$

Substitute $x = \frac{18}{7}$ into (i): $y = 2\left(\frac{18}{7}\right) - 1 = \frac{36}{7} - 1 = \frac{36}{7} - \frac{7}{7} = \frac{29}{7}$

$\therefore \quad x = \frac{18}{7}, \quad y = \frac{29}{7}$

(c) $\begin{cases} 4x + y = 11 \\ 2x + 5y = 23 \end{cases}$

Solution:

From equation (i): $y = 11 - 4x$ ---- (i)

Substitute (i) into (ii):

$2x + 5(11 - 4x) = 23$

$2x + 55 - 20x = 23$

$-18x + 55 = 23$

$-18x = 23 - 55 = -32$

$x = \frac{-32}{-18} = \frac{16}{9}$

Substitute $x = \frac{16}{9}$ into (i): $y = 11 - 4\left(\frac{16}{9}\right) = 11 - \frac{64}{9} = \frac{99}{9} - \frac{64}{9} = \frac{35}{9}$

$\therefore \quad x = \frac{16}{9}, \quad y = \frac{35}{9}$

(d) $\begin{cases} 5x - 2y = 4 \\ 3x + y = 13 \end{cases}$

Solution:

From equation (ii): $y = 13 - 3x$ ---- (i)

Substitute (i) into (ii):

$5x - 2(13 - 3x) = 4$

$5x - 26 + 6x = 4$

$11x - 26 = 4$

$11x = 30$

$x = \frac{30}{11}$

Substitute $x = \frac{30}{11}$ into (i): $y = 13 - 3\left(\frac{30}{11}\right) = 13 - \frac{90}{11} = \frac{143}{11} - \frac{90}{11} = \frac{53}{11}$

$\therefore \quad x = \frac{30}{11}, \quad y = \frac{53}{11}$

(e) $\begin{cases} x + 2y = 7 \\ 3x - y = 1 \end{cases}$

Solution:

From equation (i): $x = 7 - 2y$ ---- (i)

Substitute (i) into (ii):

$3(7 - 2y) - y = 1$

$21 - 6y - y = 1$

$21 - 7y = 1$

$-7y = 1 - 21 = -20$

$y = \frac{-20}{-7} = \frac{20}{7}$

Substitute $y = \frac{20}{7}$ into (i): $x = 7 - 2\left(\frac{20}{7}\right) = 7 - \frac{40}{7} = \frac{49}{7} - \frac{40}{7} = \frac{9}{7}$

$\therefore \quad x = \frac{9}{7}, \quad y = \frac{20}{7}$

SOLVE EACH QUESTION STEP-BY-STEP

1. Solve the quadratic equation: $3x^2 - 7x + 2 = 0$

Solution:
$$3x^2 - 7x + 2 = 0$$
$$3x^2 - 6x - x + 2 = 0$$
$$3x(x-2) - 1(x-2) = 0$$
$$(x-2)(3x-1) = 0$$
$$x - 2 = 0 \quad \text{or} \quad 3x - 1 = 0$$
$$x = 2 \quad \text{or} \quad 3x = 1$$
$$x = 2 \quad \text{or} \quad x = \frac{1}{3}$$
$$\boxed{x = 2 \text{ or } x = \frac{1}{3}}$$

2. If $(2x-3)^2 = 49$, find the value of x.

Solution:
$$(2x-3)^2 = 49$$
$$2x - 3 = \pm\sqrt{49}$$
$$2x - 3 = \pm 7$$

If $2x - 3 = 7$
$$2x = 7 + 3$$
$$2x = 10$$
$$x = 5$$

If $2x - 3 = -7$
$$2x = -7 + 3$$
$$2x = -4$$
$$x = -2$$

$$\boxed{x = 5 \text{ or } x = -2}$$

3. Simplify: $(2a^2b)^3 \times (3ab^2)^2$

Solution:
$$(2a^2b)^3 \times (3ab^2)^2$$
$$= 2^3(a^2)^3b^3 \times 3^2a^2(b^2)^2$$
$$= 2^3a^{2\times3}b^3 \times 3^2a^2b^{2\times2}$$
$$= 8a^6b^3 \times 9a^2b^4$$
$$= (8\times9)\,a^{6+2}\,b^{3+4}$$
$$= \boxed{72a^8b^7}$$

4. Factorise completely: $6x^2 - 11x + 3$

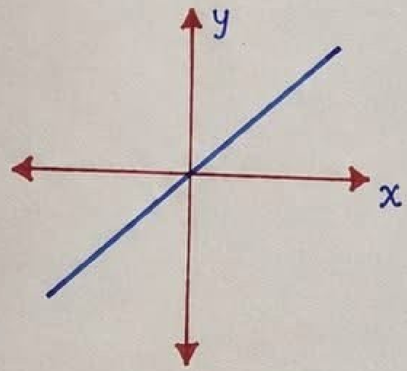
Solution:
$$6x^2 - 11x + 3$$
$$= 6x^2 - 9x - 2x + 3$$
$$= 3x(2x-3) - 1(2x-3)$$
$$= (3x-1)(2x-3)$$
$$\boxed{(3x-1)(2x-3)}$$

5. Find the gradient and y-intercept of the line: $2y = 4x - 6$

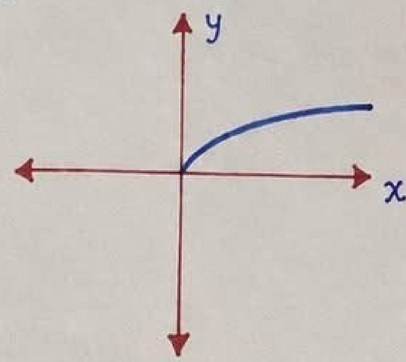
Solution:
$$2y = 4x - 6$$
Divide both sides by 2:
$$y = 2x - 3$$
Compare with $y = mx + c$
$$m = 2 \text{ (gradient)}$$
$$c = -3 \text{ (y-intercept)}$$
$$\boxed{\text{Gradient} = 2,\ \text{y-intercept} = -3}$$

GRAPHS OF FUNCTIONS

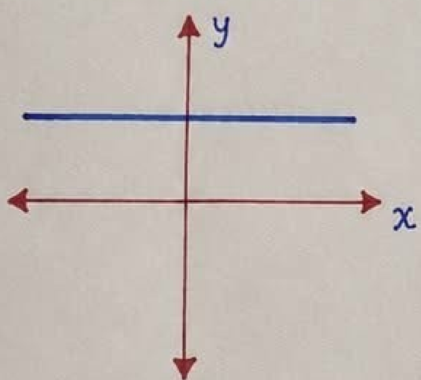
Identity: $f(x) = x$



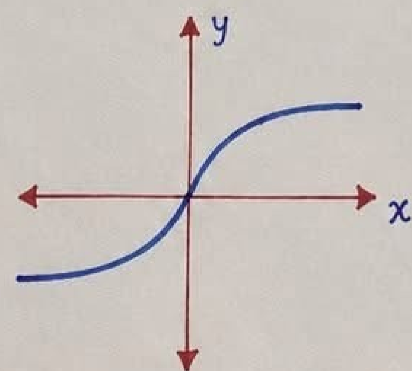
Square root: $f(x) = \sqrt{x}$



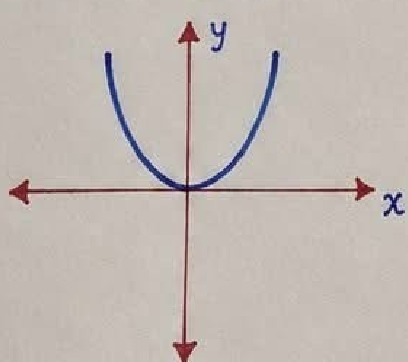
Constant: $f(x) = 2$



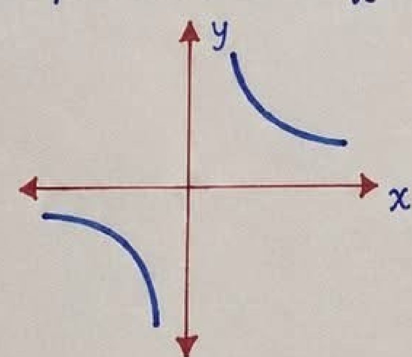
Cube root: $f(x) = \sqrt[3]{x}$



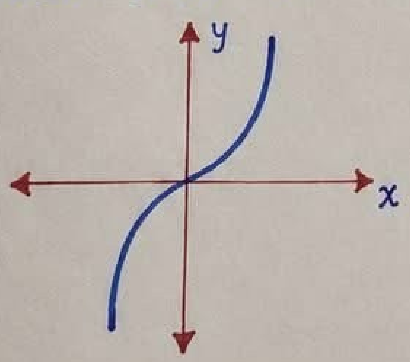
Quadratic: $f(x) = x^2$



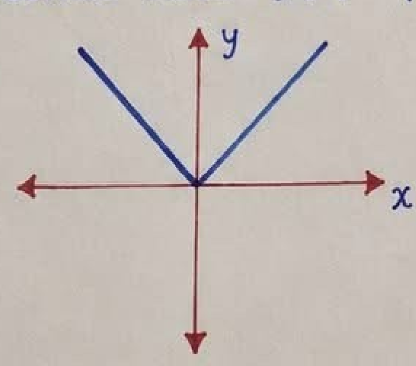
Reciprocal: $f(x) = \frac{1}{x}$



Cubic: $f(x) = x^3$



Absolute Value: $f(x) = |x|$



Solve the following simultaneous linear equations

a) $2x + 3y = 13$
$x - y = 1$

Solution:
From $x - y = 1$,
$x = 1 + y$ — ①
Substitute into $2x + 3y = 13$
$2(1 + y) + 3y = 13$
$2 + 2y + 3y = 13$
$5y = 13 - 2 = 11$
$y = \frac{11}{5}$
From ①, $x = 1 + \frac{11}{5} = \frac{16}{5}$

Solution: $\left(\frac{16}{5}, \frac{11}{5}\right)$ ✓

b) $3x - 2y = 4$
$x + 2y = 10$

Solution:
From $x + 2y = 10$,
$x = 10 - 2y$ — ①
Substitute into $3x - 2y = 4$
$3(10 - 2y) - 2y = 4$
$30 - 6y - 2y = 4$
$-8y = 4 - 30 = -26$
$y = \frac{-26}{-8} = \frac{13}{4}$
From ①, $x = 10 - 2\left(\frac{13}{4}\right) = 10 - \frac{13}{2} = \frac{7}{2}$

Solution: $\left(\frac{7}{2}, \frac{13}{4}\right)$ ✓

c) $4x + y = 11$
$2x - 3y = -7$

Solution:
From $4x + y = 11$,
$y = 11 - 4x$ — ①
Substitute into $2x - 3y = -7$
$2x - 3(11 - 4x) = -7$
$2x - 33 + 12x = -7$
$14x = -7 + 33 = 26$
$x = \frac{26}{14} = \frac{13}{7}$
From ①, $y = 11 - 4\left(\frac{13}{7}\right) = 11 - \frac{52}{7} = \frac{25}{7}$

Solution: $\left(\frac{13}{7}, \frac{25}{7}\right)$ ✓

d) $5x - y = 14$
$3x + y = 6$

Solution:
Add the two equations:
$(5x - y) + (3x + y) = 14 + 6$
$8x = 20$
$x = \frac{20}{8} = \frac{5}{2}$
Substitute into $3x + y = 6$
$3\left(\frac{5}{2}\right) + y = 6$
$\frac{15}{2} + y = 6$
$y = 6 - \frac{15}{2} = \frac{12}{2} - \frac{15}{2} = -\frac{3}{2}$

Solution: $\left(\frac{5}{2}, -\frac{3}{2}\right)$ ✓

e) $6x + 2y = 22$
$x - y = 5$

Solution:
From $x - y = 5$,
$x = 5 + y$ — ①
Substitute into $6x + 2y = 22$
$6(5 + y) + 2y = 22$
$30 + 6y + 2y = 22$
$8y = 22 - 30 = -8$
$y = \frac{-8}{8} = -1$
From ①, $x = 5 + (-1) = 4$

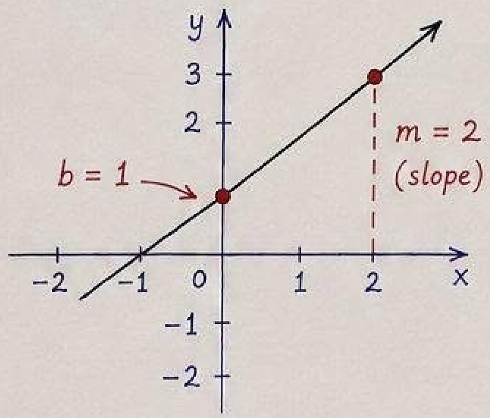
Solution: $(4, -1)$ ✓

GRAPHS OF FUNCTIONS

1.) LINEAR FUNCTION

$f(x) = mx + b$

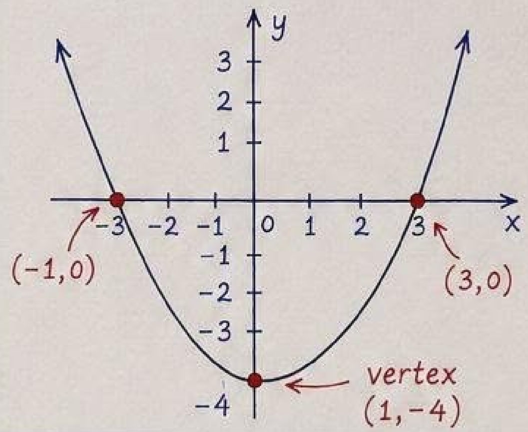
ex. $f(x) = 2x + 1$



2.) QUADRATIC FUNCTION

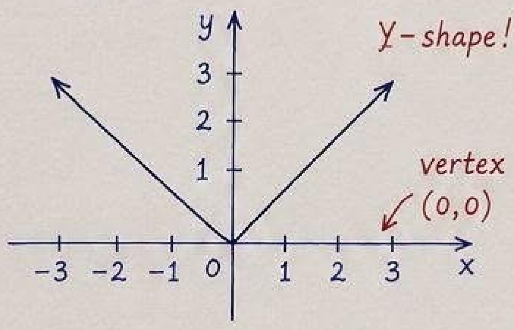
$f(x) = ax^2 + bx + c$

ex. $f(x) = x^2 - 2x - 3$



3.) ABSOLUTE VALUE FUNCTION

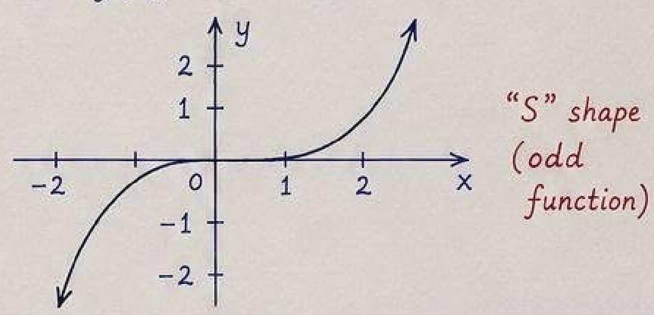
$f(x) = |x|$



4.) CUBIC FUNCTION

$f(x) = ax^3 + bx^2 + cx + d$

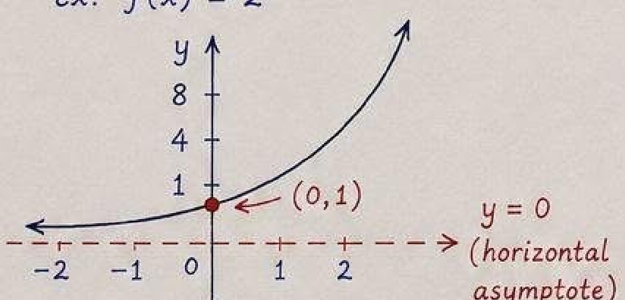
ex. $f(x) = x^3 - 3x$



5.) EXPONENTIAL FUNCTION

$f(x) = a^x \quad a > 0,\ a \neq 1$

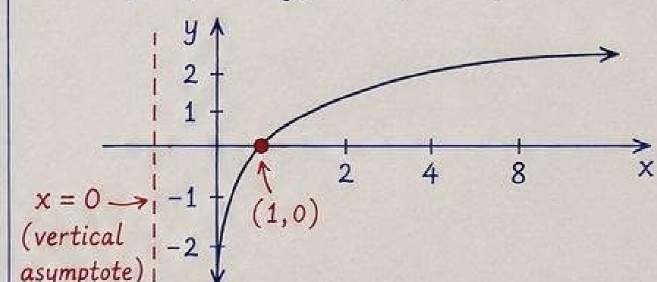
ex. $f(x) = 2^x$



6.) LOGARITHMIC FUNCTION

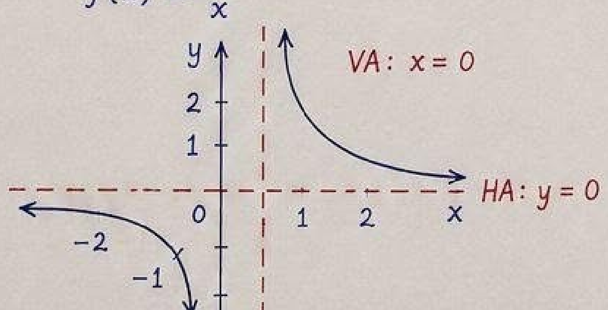
$f(x) = \log_a x \quad a > 0,\ a \neq 1$

ex. $f(x) = \log_2 x \quad (a > 1)$



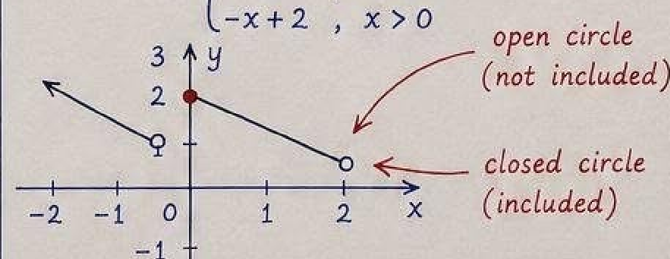
7.) RATIONAL FUNCTION

$f(x) = \frac{1}{x}$



8.) PIECEWISE FUNCTION

$$f(x) = \begin{cases} x + 2, & x < 0 \\ 2, & x = 0 \\ -x + 2, & x > 0 \end{cases}$$



Note: All graphs are shown for typical examples and may vary with different values.

Topic Box: LOGARITHMIC EQUATION

(a) $\log_2(x+3) = 4$

$$\begin{aligned}
\log_2(x+3) &= 4 \\
x + 3 &= 2^4 \\
x + 3 &= 16 \\
x &= 16 - 3 \\
x &= \underline{\underline{13}}
\end{aligned}$$

(b) $\log_3(2x-1) = 3$

$$\begin{aligned}
\log_3(2x-1) &= 3 \\
2x - 1 &= 3^3 \\
2x - 1 &= 27 \\
2x &= 27 + 1 \\
2x &= 28 \\
x &= \frac{28}{2} \\
x &= \underline{\underline{14}}
\end{aligned}$$

(c) $\log_5(x-2) = \log_5 7$

$$\begin{aligned}
\log_5(x-2) &= \log_5 7 \\
x - 2 &= 7 \\
x &= 7 + 2 \\
x &= \underline{\underline{9}}
\end{aligned}$$

(d) $2\log_4 x = \log_4(3x-4)$

$$\begin{aligned}
2\log_4 x &= \log_4(3x-4) \\
\log_4 x^2 &= \log_4(3x-4) \\
x^2 &= 3x - 4 \\
x^2 - 3x + 4 &= 0
\end{aligned}$$

For $x^2 - 3x + 4 = 0$,

$D = (-3)^2 - 4(1)(4) = 9 - 16 = -7 < 0$

$\underline{\underline{\text{No real solution.}}}$

(e) $\log_2(x+1) + \log_2(x-1) = 3$

$$\begin{aligned}
\log_2(x+1) + \log_2(x-1) &= 3 \\
\log_2((x+1)(x-1)) &= 3 \\
\log_2(x^2-1) &= 3 \\
x^2 - 1 &= 2^3 \\
x^2 - 1 &= 8 \\
x^2 &= 9 \\
x &= \underline{\pm 3}
\end{aligned}$$

Domain: $x+1 > 0$ and $x-1 > 0 \Rightarrow x > 1$

Hence, $x = \underline{\underline{3}}$

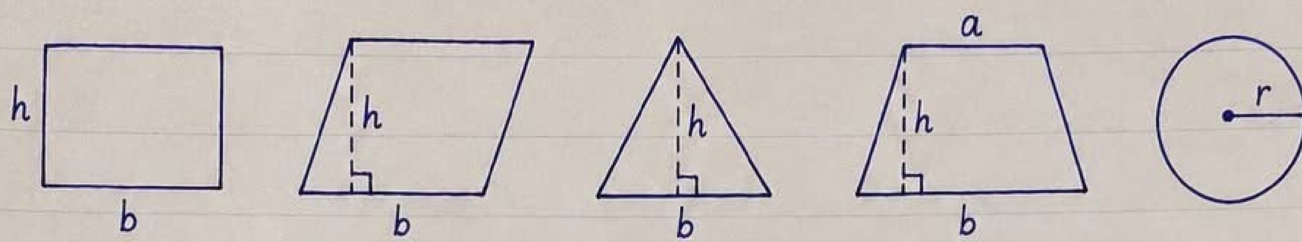
LAWS OF EXPONENTS

Let a and b be real numbers, $a \neq 0$, $b \neq 0$, and m and n be integers.

LAW	RULE	EXAMPLE	NOTES
1. Product of Powers	$a^m \cdot a^n = a^{m+n}$	$2^3 \cdot 2^4 = 2^{3+4} = 2^7 = \boxed{128}$	• When multiplying powers with the same base, add the exponents.
2. Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$	$\frac{5^6}{5^2} = 5^{6-2} = 5^4 = \boxed{625}$	• When dividing powers with the same base, subtract the exponents.
3. Power of a Power	$(a^m)^n = a^{mn}$	$(3^2)^4 = 3^{2 \cdot 4} = 3^8 = \boxed{6561}$	• When raising a power to another power, multiply the exponents.
4. Power of a Product	$(ab)^n = a^n b^n$	$(2x)^3 = 2^3 x^3 = \boxed{8x^3}$	• When raising a product to a power, raise each factor to that power.
5. Power of a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $(b \neq 0)$	$\left(\frac{3x}{2y}\right)^2 = \frac{(3x)^2}{(2y)^2} = \boxed{\frac{9x^2}{4y^2}}$	• When raising a quotient to a power, raise both the numerator and denominator to that power.
6. Zero Exponent	$a^0 = 1$ $(a \neq 0)$	$7^0 = \boxed{1}$ $(-5)^0 = \boxed{1}$	• Any nonzero number raised to the zero power is 1.
7. Negative Exponent	$a^{-n} = \frac{1}{a^n}$ $(a \neq 0)$	$2^{-3} = \frac{1}{2^3} = \boxed{\frac{1}{8}}$	• A negative exponent means take the reciprocal and make the exponent positive.
8. Negative Exponent of a Quotient	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $(a \neq 0, b \neq 0)$	$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \boxed{\frac{9}{4}}$	• Take the reciprocal of the base and make the exponent positive.
9. Fractional Exponent	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$ $(a > 0,\ n > 0)$	$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = \boxed{4}$	• A fractional exponent means a root. $a^{\frac{1}{n}} = \sqrt[n]{a}$
10. Equality of Bases	If $a^m = a^n$, then $m = n$ $(a > 0,\ a \neq 1)$	If $4^x = 4^7$, then $\boxed{x = 7}$	• If the bases are equal (and not 1), the exponents must be equal.

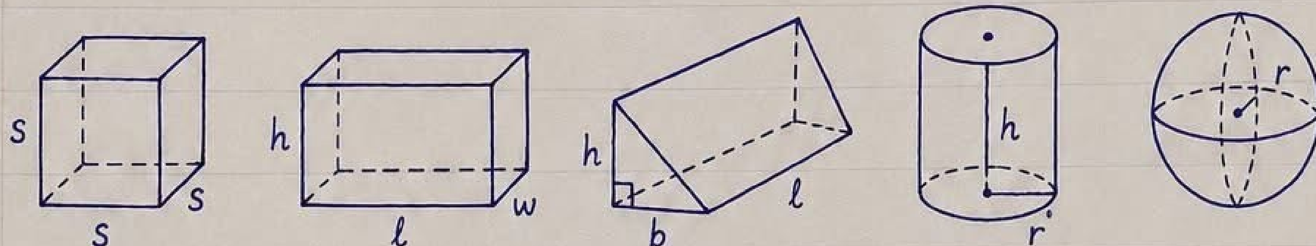
Geometry Formulas

1. Area



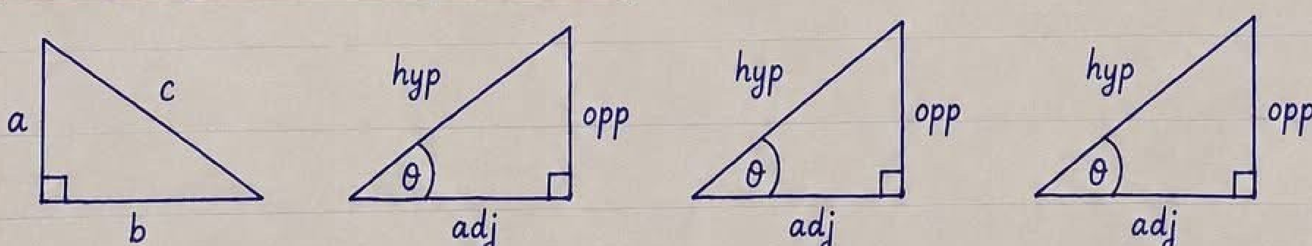
Rectangle	Parallelogram	Triangle	Trapezium	Circle
$A = bh$	$A = bh$	$A = \frac{1}{2} bh$	$A = \frac{1}{2}(a+b)h$	$A = \pi r^2$

2. Volume



Cube	Cuboid	Triangular Prism	Cylinder	Sphere
$V = s^3$	$V = lwh$	$V = \frac{1}{2} bhl$	$V = \pi r^2 h$	$V = \frac{4}{3}\pi r^3$

3. Pythagoras and Trigonometry



Pythagoras Theorem	Sine	Cosine	Tangent
$a^2 + b^2 = c^2$	$\sin(\theta) = \frac{opp}{hyp}$	$\cos(\theta) = \frac{adj}{hyp}$	$\tan(\theta) = \frac{opp}{adj}$

Simplify the following surds

Topic Box: RATIONALIZATION OF SURDS

(a) $\frac{1}{\sqrt{2}}$

Solution:

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{(\sqrt{2})^2} = \frac{\sqrt{2}}{2}$$

(b) $\frac{3}{\sqrt{5}}$

Solution:

$$\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{(\sqrt{5})^2} = \frac{3\sqrt{5}}{5}$$

(c) $\frac{2}{\sqrt{3}-1}$

Solution:

$$\frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{2(\sqrt{3}+1)}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{2(\sqrt{3}+1)}{3-1} = \frac{2(\sqrt{3}+1)}{2} = \sqrt{3}+1$$

(d) $\frac{5}{2+\sqrt{3}}$

Solution:

$$\frac{5}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{5(2-\sqrt{3})}{(2)^2-(\sqrt{3})^2}$$

$$= \frac{5(2-\sqrt{3})}{4-3} = \frac{5(2-\sqrt{3})}{1} = 5(2-\sqrt{3})$$

(e) $\frac{1}{\sqrt{7}+\sqrt{2}}$

Solution:

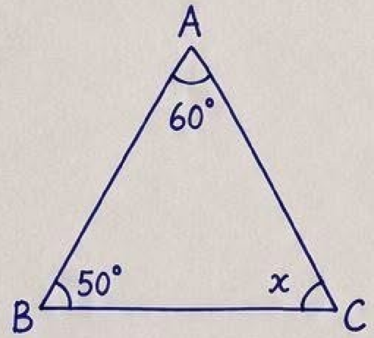
$$\frac{1}{\sqrt{7}+\sqrt{2}} \times \frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}-\sqrt{2}} = \frac{\sqrt{7}-\sqrt{2}}{(\sqrt{7})^2-(\sqrt{2})^2}$$

$$= \frac{\sqrt{7}-\sqrt{2}}{7-2} = \frac{\sqrt{7}-\sqrt{2}}{5}$$

Solve for the unknown angle in a triangle

Topic Box: UNKNOWN ANGLE IN A TRIANGLE

(a)



In a triangle, the sum of angles = 180°

$60° + 50° + x = 180°$

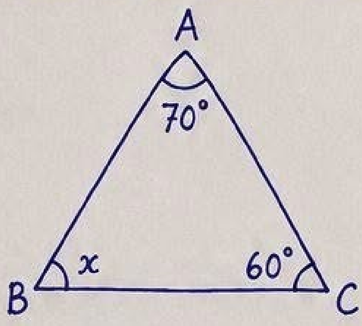
$110° + x = 180°$

$x = 180° - 110°$

$x = \underline{\underline{70°}}$

✓

(b)



In a triangle, the sum of angles = 180°

$x + 70° + 60° = 180°$

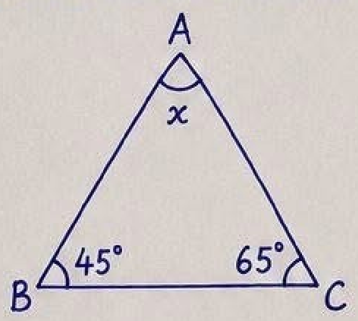
$x + 130° = 180°$

$x = 180° - 130°$

$x = \underline{\underline{50°}}$

✓

(c)



In a triangle, the sum of angles = 180°

$x + 45° + 65° = 180°$

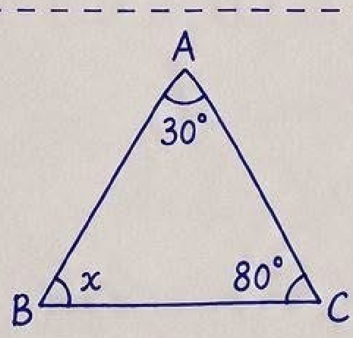
$x + 110° = 180°$

$x = 180° - 110°$

$x = \underline{\underline{70°}}$

✓

(d)



In a triangle, the sum of angles = 180°

$x + 30° + 80° = 180°$

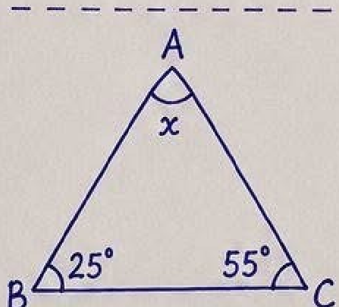
$x + 110° = 180°$

$x = 180° - 110°$

$x = \underline{\underline{70°}}$

✓

(e)



In a triangle, the sum of angles = 180°

$x + 25° + 55° = 180°$

$x + 80° = 180°$

$x = 180° - 80°$

$x = \underline{\underline{100°}}$

✓

Solve the following quadratic equation by formula method

QUADRATIC EQUATION BY FORMULA METHOD

(a) $x^2 + 5x + 6 = 0$

$a = 1, \quad b = 5, \quad c = 6$

$b^2 - 4ac = 5^2 - 4(1)(6) = 25 - 24 = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{1}}{2(1)} = \frac{-5 \pm 1}{2}$$

$$x = \frac{-5+1}{2} = \frac{-4}{2} = -2 \quad \text{or} \quad x = \frac{-5-1}{2} = \frac{-6}{2} = -3$$

$\underline{\underline{x = -2}}$ or $\underline{\underline{x = -3}}$ ✓

(b) $2x^2 - 3x - 2 = 0$

$a = 2, \quad b = -3, \quad c = -2$

$b^2 - 4ac = (-3)^2 - 4(2)(-2) = 9 + 16 = 25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{25}}{2(2)} = \frac{3 \pm 5}{4}$$

$$x = \frac{3+5}{4} = \frac{8}{4} = 2 \quad \text{or} \quad x = \frac{3-5}{4} = \frac{-2}{4} = -\frac{1}{2}$$

$\underline{\underline{x = 2}}$ or $\underline{\underline{x = -\frac{1}{2}}}$ ✓

(c) $3x^2 + 2x - 8 = 0$

$a = 3, \quad b = 2, \quad c = -8$

$b^2 - 4ac = 2^2 - 4(3)(-8) = 4 + 96 = 100$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{100}}{2(3)} = \frac{-2 \pm 10}{6}$$

$$x = \frac{-2+10}{6} = \frac{8}{6} = \frac{4}{3} \quad \text{or} \quad x = \frac{-2-10}{6} = \frac{-12}{6} = -2$$

$\underline{\underline{x = 4/3}}$ or $\underline{\underline{x = -2}}$ ✓

(d) $4x^2 - 4x - 3 = 0$

$a = 4, \quad b = -4, \quad c = -3$

$b^2 - 4ac = (-4)^2 - 4(4)(-3) = 16 + 48 = 64$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{64}}{2(4)} = \frac{4 \pm 8}{8}$$

$$x = \frac{4+8}{8} = \frac{12}{8} = \frac{3}{2} \quad \text{or} \quad x = \frac{4-8}{8} = \frac{-4}{8} = -\frac{1}{2}$$

$\underline{\underline{x = 3/2}}$ or $\underline{\underline{x = -1/2}}$ ✓

(e) $5x^2 + 7x + 1 = 0$

$a = 5, \quad b = 7, \quad c = 1$

$b^2 - 4ac = 7^2 - 4(5)(1) = 49 - 20 = 29$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{29}}{2(5)} = \frac{-7 \pm \sqrt{29}}{10}$$

$$x = \frac{-7 + \sqrt{29}}{10} \quad \text{or} \quad x = \frac{-7 - \sqrt{29}}{10}$$ ✓

Simplify the following algebraic fractions.

SIMPLIFICATION OF ALGEBRAIC FRACTIONS

(a) $$\frac{x^2-9}{x^2-3x} = \frac{x^2-9}{x^2-3x} = \frac{(x-3)(x+3)}{x(x-3)}$$

$$= \frac{x+3}{x}$$

(b) $$\frac{2x+6}{x^2+3x} = \frac{2x+6}{x^2+3x} = \frac{2(x+3)}{x(x+3)}$$

$$= \frac{2}{x}$$

(c) $$\frac{x^2-4x}{x^2-16} = \frac{x^2-4x}{x^2-16} = \frac{x(x-4)}{(x-4)(x+4)}$$

$$= \frac{x}{x+4}$$

(d) $$\frac{x^2+5x+6}{x^2+3x+2} = \frac{x^2+5x+6}{x^2+3x+2} = \frac{(x+2)(x+3)}{(x+1)(x+2)}$$

$$= \frac{x+3}{x+1}$$

(e) $$\frac{3x^2-12x}{6x^2-24} = \frac{3x^2-12x}{6x^2-24} = \frac{3x(x-4)}{6(x^2-4)}$$

$$= \frac{3x(x-4)}{6(x-2)(x+2)} = \frac{x}{2(x+2)}$$

IMPORTANT MATHEMATICAL VALUES

EXAM SHORTCUT TABLES

* These values are very useful in calculations, approximations and problem solving.

S. No.	VALUE / CONSTANT	SYMBOL	VALUE (Approx.)	NOTES / USE
1.	Pi (Circle Constant)	π	3.1415926535 $\approx$ 3.1416	Used in area, circumference, of circle, sphere, etc.
2.	Euler's Number	e	2.718281828 $\approx$ 2.71828	Used in calculus, logarithms, exponential growth, etc.
3.	Golden Ratio	ϕ (phi)	1.6180339887 $\approx$ 1.618	Appears in geometry, design, Fibonacci series, etc.
4.	Square Root of 2	$\sqrt{2}$	1.414213562 $\approx$ 1.4142	Used in Pythagoras theorem, diagonals of squares, etc.
5.	Square Root of 3	$\sqrt{3}$	1.732050808 $\approx$ 1.7321	Used in equilateral triangles, trigonometry, etc.
6.	Square Root of 5	$\sqrt{5}$	2.236067977 $\approx$ 2.2361	Used in geometry, distances, diagonals, etc.
7.	Square Root of 10	$\sqrt{10}$	3.162277660 $\approx$ 3.1623	Used in 3D geometry, distances, etc.
8.	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0.707106781 $\approx$ 0.7071	Used in trigonometry (45°), normalization, etc.
9.	Degree to Radian	$\frac{\pi}{180}$	0.0174532925 $\approx$ 0.01745	Multiply degrees by this to convert into radians.
10.	Radian to Degree	$\frac{180}{\pi}$	57.29577951 $\approx$ 57.296	Multiply radians by this to convert into degrees.
11.	Natural Log of 2	$\ln 2$	0.693147181 $\approx$ 0.6931	Used in logarithms and calculus.
12.	Natural Log of 10	$\ln 10$	2.302585093 $\approx$ 2.3026	Used in logarithms, scientific calculations.
13.	Common Log of e	$\log_{10} e$	0.434294482 $\approx$ 0.4343	Change of base in logarithms.
14.	Sine of 30°	$\sin 30°$	0.5	Basic trigonometric value.
15.	Cosine of 60°	$\cos 60°$	0.5	Basic trigonometric value.
16.	Sine of 45°	$\sin 45°$	0.707106781 $\approx$ 0.7071	$= 1/\sqrt{2}$
17.	Cosine of 45°	$\cos 45°$	0.707106781 $\approx$ 0.7071	$= 1/\sqrt{2}$
18.	Sine of 60°	$\sin 60°$	0.866025404 $\approx$ 0.8660	$= \sqrt{3}/2$
19.	Cosine of 30°	$\cos 30°$	0.866025404 $\approx$ 0.8660	$= \sqrt{3}/2$
20.	$\frac{1}{\pi}$	$\frac{1}{\pi}$	0.318309886 $\approx$ 0.3183	Used in many shortcut calculations.

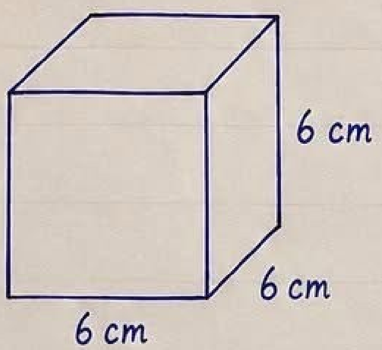
QUICK TIPS

* Learn these values for fast calculation.
* Use approximations as per the required accuracy.
* These values are useful in every area of Mathematics.

Topic Box: VOLUME OF CUBES AND CUBOIDS

Find the volume of the following cubes and cuboids

(a)



Solution:

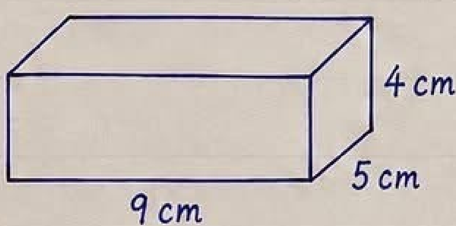
Volume of a cube $= s^3$

$= 6^3$

$= 6 \times 6 \times 6$

$= 216 \text{ cm}^3$ ✓

(b)



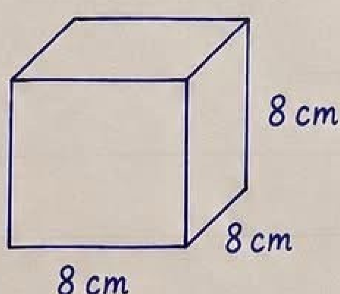
Solution:

Volume of a cuboid $= \ell \times w \times h$

$= 9 \times 5 \times 4$

$= 180 \text{ cm}^3$ ✓

(c)



Solution:

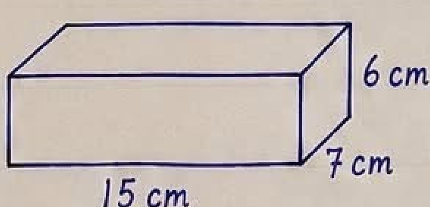
Volume of a cube $= s^3$

$= 8^3$

$= 8 \times 8 \times 8$

$= 512 \text{ cm}^3$ ✓

(d)



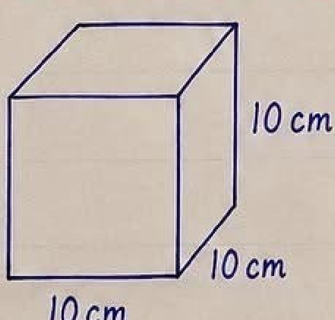
Solution:

Volume of a cuboid $= \ell \times w \times h$

$= 15 \times 7 \times 6$

$= 630 \text{ cm}^3$ ✓

(e)



Solution:

Volume of a cube $= s^3$

$= 10^3$

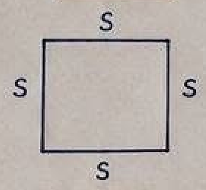
$= 10 \times 10 \times 10$

$= 1000 \text{ cm}^3$ ✓

PERIMETER, AREA AND VOLUME

PERIMETER — the total distance around the outside of a shape.

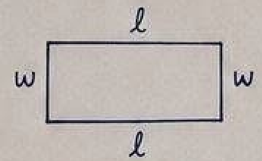
SQUARE



$P = 4s$

where s = side

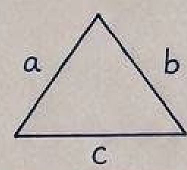
RECTANGLE



$P = 2(\ell + w)$

where ℓ = length
w = width

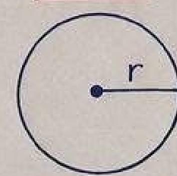
TRIANGLE



$P = a + b + c$

where a, b, c are the sides

CIRCLE

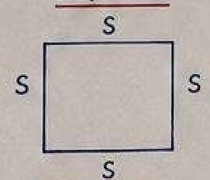


$P = 2\pi r$

where r = radius
($\pi \approx 3.14$)

AREA — the amount of space inside a two-dimensional shape.

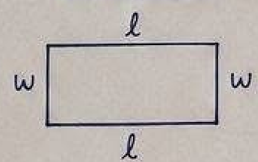
SQUARE



$A = s^2$

where s = side

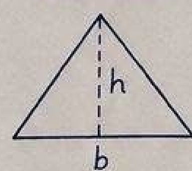
RECTANGLE



$A = \ell \times w$

where ℓ = length
w = width

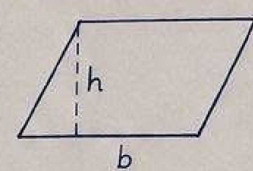
TRIANGLE



$A = \frac{1}{2} bh$

where b = base
h = height

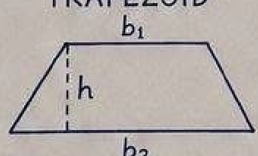
PARALLELOGRAM



$A = bh$

where b = base
h = height

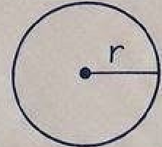
TRAPEZOID



$A = \frac{1}{2}(b_1 + b_2)h$

where b_1, b_2 = bases
h = height

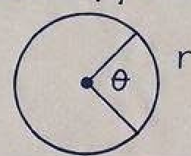
CIRCLE



$A = \pi r^2$

where r = radius
($\pi \approx 3.14$)

SECTOR (of a circle)

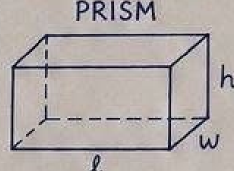


$A = \frac{\theta}{360°} \pi r^2$

where θ = central angle in degrees

VOLUME — the amount of space inside a three-dimensional solid.

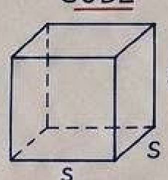
RECTANGULAR PRISM



$V = \ell w h$

where ℓ = length
w = width
h = height

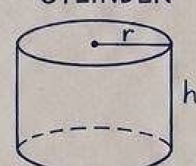
CUBE



$V = s^3$

where s = side

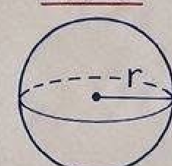
CYLINDER



$V = \pi r^2 h$

where r = radius
h = height
($\pi \approx 3.14$)

SPHERE



$V = \frac{4}{3} \pi r^3$

where r = radius
($\pi \approx 3.14$)

BASIC TRIGONOMETRIC RATIOS

In a right triangle, the trigonometric ratios relate the measures of the acute angles to the lengths of the sides.

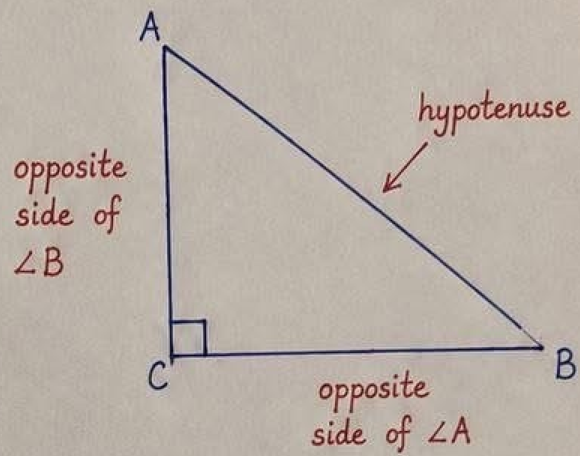
Let Δ ABC be a right triangle right-angled at C.

For angle A :

Opposite side = BC
Adjacent side = AC
Hypotenuse = AB

For angle B :

Opposite side = AC
Adjacent side = BC
Hypotenuse = AB



The six trigonometric ratios :

$$\boxed{\sin A} = \frac{\text{opposite side of } \angle A}{\text{hypotenuse}} = \frac{BC}{AB}$$

$$\boxed{\cos A} = \frac{\text{adjacent side of } \angle A}{\text{hypotenuse}} = \frac{AC}{AB}$$

$$\boxed{\tan A} = \frac{\text{opposite side of } \angle A}{\text{adjacent side of } \angle A} = \frac{BC}{AC}$$

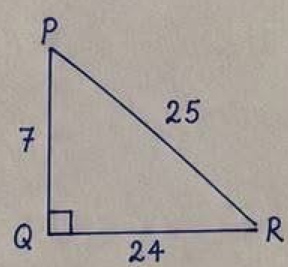
$$\boxed{\sin B} = \frac{\text{opposite side of } \angle B}{\text{hypotenuse}} = \frac{AC}{AB}$$

$$\boxed{\cos B} = \frac{\text{adjacent side of } \angle B}{\text{hypotenuse}} = \frac{BC}{AB}$$

$$\boxed{\tan B} = \frac{\text{opposite side of } \angle B}{\text{adjacent side of } \angle B} = \frac{AC}{BC}$$

Examples :

1. In Δ PQR, right-angled at Q. PQ = 7, QR = 24, PR = 25 Find the six trigonometric ratios of angle P.

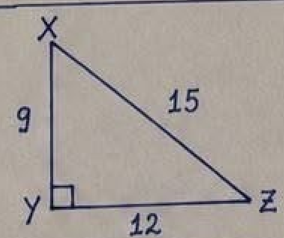


$\sin P = \frac{24}{25}$ $\quad$ $\sin R = \frac{7}{25}$

$\cos P = \frac{7}{25}$ $\quad$ $\cos R = \frac{24}{25}$

$\tan P = \frac{24}{7}$ $\quad$ $\tan R = \frac{7}{24}$

2. In Δ XYZ, right-angled at Y. XY = 9, YZ = 12, XZ = 15 Find the six trigonometric ratios of angle X.

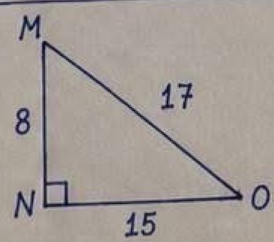


$\sin X = \frac{12}{15}$ $\quad$ $\sin Z = \frac{9}{15}$

$\cos X = \frac{9}{15}$ $\quad$ $\cos Z = \frac{12}{15}$

$\tan X = \frac{12}{9}$ $\quad$ $\tan Z = \frac{9}{12}$

3. In Δ MNO, right-angled at N. MN = 8, NO = 15, MO = 17 Find the six trigonometric ratios of angle M.



$\sin M = \frac{15}{17}$ $\quad$ $\sin O = \frac{8}{17}$

$\cos M = \frac{8}{17}$ $\quad$ $\cos O = \frac{15}{17}$

$\tan M = \frac{15}{8}$ $\quad$ $\tan O = \frac{8}{15}$

NUMBER SYSTEM

Important Numbers :

1. Natural numbers → 1, 2, 3, 4, 5, 6, ...
2. Even numbers → 2, 4, 6, 8, 10, 12, ...
3. Odd numbers → 1, 3, 5, 7, 9, 11, ...
4. Whole numbers → 0, 1, 2, 3, 4, 5, ...
5. Integer numbers → ..., −3, −2, −1, 0, 1, 2, 3, ...
6. Prime numbers → 2, 3, 5, 7, 11, ...
7. Composite numbers → 4, 6, 8, 9, 10, 12, ...
8. Co-prime numbers → (5, 7), (2, 3), (8, 15)
9. Rational numbers → $0, \frac{1}{2}, -\frac{3}{1}, 5, -1, \sqrt{4}$
10. Irrational numbers → $\pi, \sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}$
11. Real numbers = Rational + Irrational

    Examples: $2, -5, 0, \frac{1}{2}, \sqrt{2}, \pi$

Smallest Numbers :

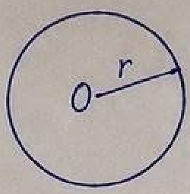
- Smallest prime number = 2
- Smallest even prime number = 2
- Smallest composite number = 4
- Smallest whole number = 0
- Smallest natural number = 1

Other Important Value :

- π (Pi) $\approx \frac{22}{7}$ or 3.14159

CIRCLE FORMULAS

① Circle :

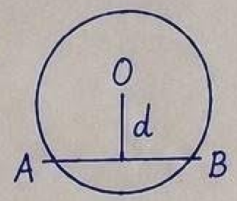


Circumference $(C) = 2\pi r$

Area $(A) = \pi r^2$

where,
r = radius

② Chord :

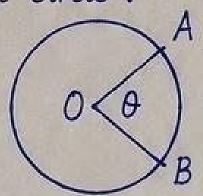


Length of chord $AB = 2\sqrt{r^2 - d^2}$

d = perpendicular distance from centre to the chord

where,
r = radius
d = distance from centre to chord

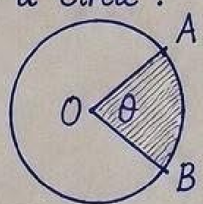
③ Arc of a Circle :



Length of arc $AB = \dfrac{\theta}{360^\circ} \times 2\pi r$

where,
r = radius
θ = angle in degrees

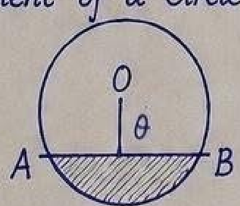
④ Sector of a Circle :



Area of sector $= \dfrac{\theta}{360^\circ} \times \pi r^2$

where,
r = radius
θ = angle in degrees

⑤ Segment of a Circle :

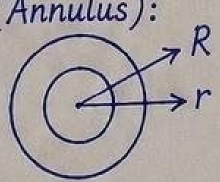


Area of segment = Area of sector − Area of ΔAOB

$= \dfrac{\theta}{360^\circ} \times \pi r^2 - \dfrac{1}{2} r^2 \sin\theta$

where,
r = radius
θ = angle in degrees

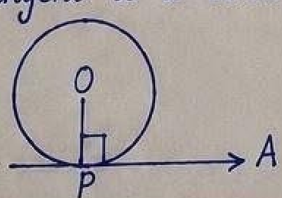
⑥ Ring (Annulus):



Area of ring $= \pi (R^2 - r^2)$

where,
R = outer radius
r = inner radius

⑦ Tangent to a Circle :



Length of tangent $PA = \sqrt{OA^2 - r^2}$

$(OP \perp PA)$

where,
OA = distance from centre O to external point A
r = radius

Important Points :

- ★ The perpendicular from the centre of a circle to a chord bisects the chord.
- ★ Equal chords of a circle are equidistant from the centre.
- ★ The angle subtended by an arc at the centre is double the angle subtended by the same arc at any point on the remaining part of the circle.

SPECIAL PRODUCTS

Special products are products of two binomials that follow special patterns. These formulas help us multiply binomials quickly.

FORMULA	EXAMPLE
1 SQUARE OF A BINOMIAL The square of a binomial is: $\boxed{(a+b)^2 = a^2 + 2ab + b^2}$ and $\boxed{(a-b)^2 = a^2 - 2ab + b^2}$	Example 1: $(x+3)^2$ $(x+3)^2 = x^2 + 2(x)(3) + 3^2$ $= x^2 + 6x + 9$ Example 2: $(2x-5)^2$ $(2x-5)^2 = (2x)^2 - 2(2x)(5) + 5^2$ $= 4x^2 - 20x + 25$
2 PRODUCT OF SUM AND DIFFERENCE OF TWO TERMS The product of the sum and the difference of two terms is: $\boxed{(a+b)(a-b) = a^2 - b^2}$	Example 1: $(x+4)(x-4)$ $(x+4)(x-4) = x^2 - 4^2$ $= x^2 - 16$ Example 2: $(3a+2)(3a-2)$ $(3a+2)(3a-2) = (3a)^2 - 2^2$ $= 9a^2 - 4$
3 SUM (OR DIFFERENCE) OF CUBES The sum of two cubes is: $\boxed{a^3 + b^3 = (a+b)(a^2 - ab + b^2)}$ and The difference of two cubes is: $\boxed{a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$	Example 1: $x^3 + 8$ $x^3 + 8 = x^3 + 2^3$ $= (x+2)(x^2 - 2x + 2^2)$ $= (x+2)(x^2 - 2x + 4)$ Example 2: $27a^3 - 1$ $27a^3 - 1 = (3a)^3 - 1^3$ $= (3a-1)((3a)^2 + (3a)(1) + 1^2)$ $= (3a-1)(9a^2 + 3a + 1)$
4 SUM (OR DIFFERENCE) OF FOURTH POWERS The sum of two fourth powers is: $\boxed{a^4 + b^4 = (a^2 + \sqrt{2}ab + b^2)(a^2 - \sqrt{2}ab + b^2)}$ and The difference of two fourth powers is: $\boxed{a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)}$	Example 1: $x^4 + 16$ $x^4 + 16 = x^4 + 2^4$ $= (x^2 + 2\sqrt{2}x + 4)(x^2 - 2\sqrt{2}x + 4)$ Example 2: $81x^4 - 16y^4$ $81x^4 - 16y^4 = (3x)^4 - (2y)^4$ $= ((3x)^2 - (2y)^2)((3x)^2 + (2y)^2)$ $= (9x^2 - 4y^2)(9x^2 + 4y^2)$

REMEMBER: These formulas work only when the expressions are in the exact form shown. Make sure to identify the pattern before applying the formula.