ROMAN NUMERALS (COMPLETE NOTES)

→ Roman numerals are a system of numbers used in ancient Rome. They are made up of combinations of seven letters.

1. BASIC SYMBOLS

Symbol	Value	Symbol	Value
I	1	C	100
V	5	D	500
X	10	M	1000
L	50		

2. RULES TO REMEMBER

- A symbol written after a greater value is added.
  Ex: VI = 5 + 1 = 6
- A symbol written before a greater value is subtracted.
  Ex: IV = 5 − 1 = 4
- A symbol cannot be repeated more than three times in a row.
  Ex: III = 3 , XXX = 30 , CCC = 300
- V, L and D are never repeated.
- Only I, X, C and M can be repeated.
- In subtraction, a smaller value can be placed before only the next greater value.

3. SUBTRACTION RULES (IMPORTANT)

Can be Subtracted	From	Example	Can be Subtracted	From	Example
I (1)	V (5)	IV = 4	C (100)	D (500)	CD = 400
I (1)	X (10)	IX = 9	C (100)	M (1000)	CM = 900
X (10)	L (50)	XL = 40	(no other subtraction allowed)	–	–
X (10)	C (100)	XC = 90			

4. ROMAN NUMERALS 1 TO 50

1 = I	11 = XI	21 = XXI	31 = XXXI	41 = XLI
2 = II	12 = XII	22 = XXII	32 = XXXII	42 = XLII
3 = III	13 = XIII	23 = XXIII	33 = XXXIII	43 = XLIII
4 = IV	14 = XIV	24 = XXIV	34 = XXXIV	44 = XLIV
5 = V	15 = XV	25 = XXV	35 = XXXV	45 = XLV
6 = VI	16 = XVI	26 = XXVI	36 = XXXVI	46 = XLVI
7 = VII	17 = XVII	27 = XXVII	37 = XXXVII	47 = XLVII
8 = VIII	18 = XVIII	28 = XXVIII	38 = XXXVIII	48 = XLVIII
9 = IX	19 = XIX	29 = XXIX	39 = XXXIX	49 = XLIX
10 = X	20 = XX	30 = XXX	40 = XL	50 = L

5. EXAMPLES

- 7 = VII
- 9 = IX
- 14 = XIV
- 19 = XIX
- 24 = XXIV
- 38 = XXXVIII
- 44 = XLIV
- 49 = XLIX

6. ROMAN NUMERALS (TENS, HUNDREDS, THOUSANDS)

TENS	HUNDREDS	THOUSANDS
10 = X	100 = C	1000 = M
20 = XX	200 = CC	2000 = MM
30 = XXX	300 = CCC	3000 = MMM
40 = XL	400 = CD	4000 = MMMM
50 = L	500 = D	
60 = LX	600 = DC	
70 = LXX	700 = DCC	
80 = LXXX	800 = DCCC	
90 = XC	900 = CM	

(Note: In modern use, repetition beyond 3 is allowed only for M)

7. IMPORTANT POINTS

✓ Write the numerals from largest to smallest value.

✓ Use subtraction rule to make the number shorter.

✓ Never repeat V, L, D.

✓ Never subtract a number from the same or smaller number.

Practice makes perfect !
Learn it well and
you will never forget.

8. LARGE NUMBERS EXAMPLES

- 1987 = MCMLXXXVII
- 2024 = MMXXIV
- 1666 = MDCLXVI
- 3999 = MMMCMXCIX
- 450 = CDL
- 944 = CMXLIV

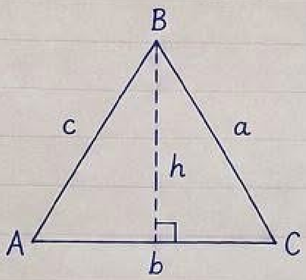
Tip : Learn the values, rules and practice daily to write any Roman numeral easily.

GEOMETRY FORMULAS (COMPLETE REVISION SHEET)

1. TRIANGLES

- Perimeter $= a + b + c$
- Area $= \frac{1}{2} bh$
- Semi-perimeter (s)

  $s = \frac{a+b+c}{2}$



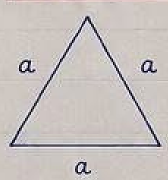
- Heron's Formula

  Area $= \sqrt{s(s-a)(s-b)(s-c)}$

- Area using Sine Forula

  Area $= \frac{1}{2}\, ab \sin C = \frac{1}{2}\, bc \sin A = \frac{1}{2}\, ca \sin B$

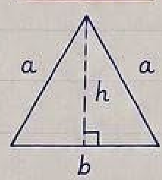
EQUILATERAL



Perimeter $= 3a$

Area $= \frac{\sqrt{3}}{4}\, a^2$

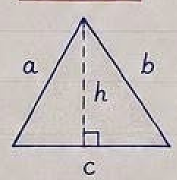
ISOSCELES



Perimeter $= 2a+b$

Area $= \frac{1}{2}\, bh$

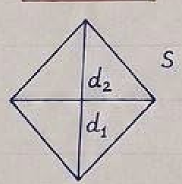
SCALENE



Perimeter $= a+b+c$

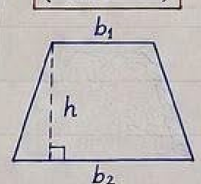
Area $= \frac{1}{2}\, bh$

RHOMBUS



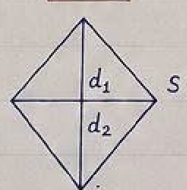
- Perimeter $= 4s$
- Area $= \frac{1}{2}\, d_1 d_2$

TRAPEZIUM (TRAPEZOID)



- Perimeter $= a+b+b_1+b_2$
- Area $= \frac{1}{2}(b_1+b_2)h$

KITE



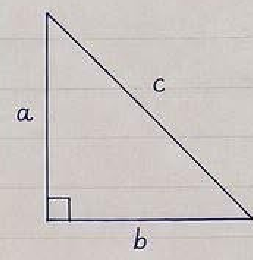
- Perimeter $= 2(s+t)$
- Area $= \frac{1}{2}\, d_1 d_2$

2. SPECIAL TRIANGLES

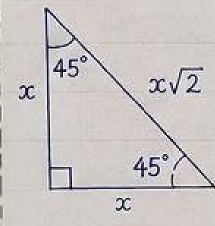
- Pythagoras Theorem

  $c^2 = a^2 + b^2$

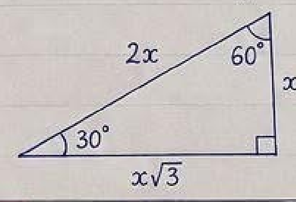
- Right Triangle



- $45° - 45° - 90°$ Triangle



- $30° - 60° - 90°$ Triangle



3. CIRCLE & POLYGONS

CIRCLE

- Circumference $= 2\pi r$
- Area $= \pi r^2$
- Arc Length $= r\theta$

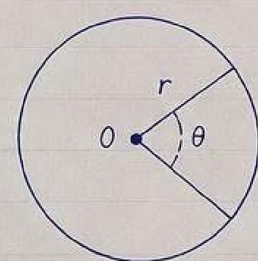
  (θ in radians)

- Area of Sector $= \frac{1}{2}\, r^2\theta$

  (θ in radians)

- Area of Segment =

  Area of Sector − Area of Triangle



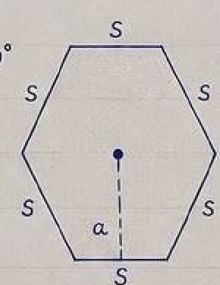
POLYGONS

- Sum of Interior Angles $= (n-2) \times 180°$
- Exterior Angle $= \frac{360°}{n}$
- Regular Polygon (n sides)

  Exterior Angle $= \frac{360°}{n}$

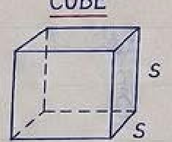
- Area of Regular Polygon $= \frac{1}{2}\, Pa$

  where P = perimeter, a = apothem



4. SOLIDS

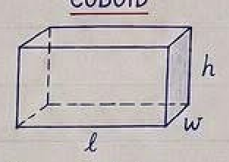
CUBE



Volume $= s^3$

TSA $= 6s^2$

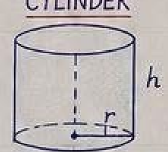
CUBOID



Volume $= \ell wh$

TSA $= 2(\ell w + \ell h + wh)$

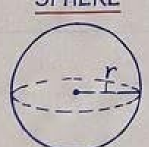
CYLINDER



Volume $= \pi r^2 h$

TSA $= 2\pi r(r+h)$

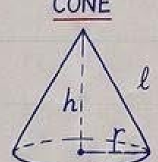
SPHERE



Volume $= \frac{4}{3}\pi r^3$

TSA $= 4\pi r^2$

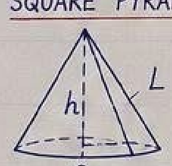
CONE



Volume $= \frac{1}{3}\pi r^2 h$

TSA $= \pi r(r+\ell)$

SQUARE PYRAMID



Volume $= \frac{1}{3}\, a^2 h$

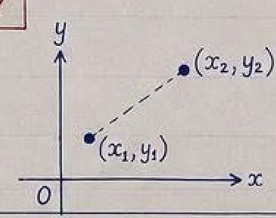
TSA $= a^2 + 2aL$

5. COORDINATE GEOMETRY

- Distance Formula

  $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

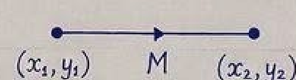
  for points (x_1, y_1) and (x_2, y_2)



- Midpoint Formula

  $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

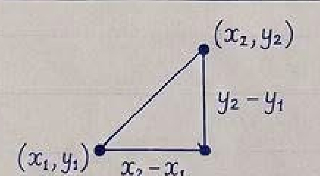
  for points (x_1, y_1) and (x_2, y_2)



- Gradient (Slope) Formula

  $m = \frac{y_2-y_1}{x_2-x_1}$, $(x_1 \neq x_2)$

  for points (x_1, y_1) and (x_2, y_2)



☆ Remember : Always label all sides, angles and heights clearly.

You can do it ! Keep Practising! ☺

ALGEBRA

Algebra is the branch of mathematics that uses letters (variables) to represent numbers and solve problems.

1. VARIABLES AND CONSTANTS

- Variable: A letter that represents a number.
  Ex: $x, y, a, b, m, n, \ldots$
- Constant: A fixed number.
  Ex: $2, -5, 7, 12, \ldots$

Examples:

- In $3x + 5$, x is a variable and $3, 5$ are constants.
- In $2a^2 - 7b + 4$, a and b are variables, $2, -7, 4$ are constants.

2. LIKE TERMS

Terms that have the same variable raised to the same power are called like terms.

Ex: $5x, -2x, 7x$ are like terms

$3y^2, -6y^2$ are like terms

Examples:

- Simplify: $4x + 3x - 7x$

$$= (4 + 3 - 7)x$$
$$= 0x = \underline{\underline{0}}$$

3. ADDING AND SUBTRACTING ALGEBRAIC EXPRESSIONS

Add or subtract like terms only.

Examples:

- $(4x + 7) + (2x - 3)$

$$= 4x + 7 + 2x - 3$$
$$= (4x + 2x) + (7 - 3)$$
$$= \underline{6x + 4}$$

Examples:

- $(5y^2 - 4y + 6) - (2y^2 + 3y - 1)$

$$= 5y^2 - 4y + 6 - 2y^2 - 3y + 1$$
$$= (5y^2 - 2y^2) + (-4y - 3y) + (6 + 1)$$
$$= \underline{3y^2 - 7y + 7}$$

4. MULTIPLICATION OF ALGEBRAIC TERMS

Multiply the coefficients and add the powers of like variables.

Examples:

- $3x \times 4x$

$$= (3 \times 4)(x \times x)$$
$$= \underline{12x^2}$$

- $-2a^2 \times 5a$

$$= (-2 \times 5)(a^2 \times a)$$
$$= \underline{-10a^3}$$

- $(2x\ y) \times (3xy^3)$

$$= (2 \times 3)(x^2 \times x)(y \times y^3)$$
$$= \underline{\underline{6x^3y^4}}$$

5. DIVISION OF ALGEBRAIC TERMS

Divide the coefficients and subtract the powers of like variables.

Examples:

- $12x^3 \div 3x$

$$= (12 \div 3)\, x^{3-1}$$
$$= \underline{4x^2}$$

- $-15a^4 \div 5a^2$

$$= (-15 \div 5)\, a^{4-2}$$
$$= \underline{-3a^2}$$

- $8x^2y^3 \div 2xy$

$$= (8 \div 2)\, x^{2-1}\, y^{3-1}$$
$$= \underline{\underline{4xy^2}}$$

*** Important Points:**

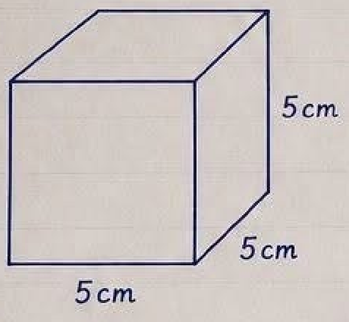
- Always work with like terms.
- Remember the rules of signs while adding, subtracting and multiplying.
- Practice more examples to improve speed and accuracy.

Practice Today, 'Excel' Tomorrow. :)

★ Find the area of the following solid shapes

AREA OF A SOLID SHAPE

(a)



Solution:

Area of a cube $= 6a^2$

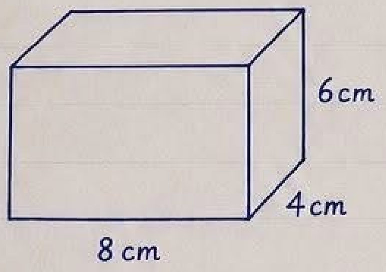
Where a = side of the cube

Here, $a = 5\text{cm}$

$$\begin{aligned} \text{Area} &= 6 \times a^2 \\ &= 6 \times 5^2 \\ &= 6 \times 25 \\ &= \underline{\underline{150 \text{ cm}^2}} \end{aligned}$$

✓

(b)



Solution:

Area of a cuboid $= 2(lw + lh + wh)$

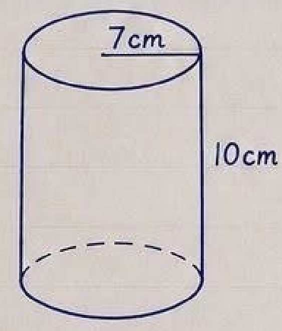
Where l = length, w = width, h = height

Here, $l = 8\text{cm}$, $w = 4\text{cm}$, $h = 6\text{cm}$

$$\begin{aligned} \text{Area} &= 2[(8 \times 4) + (8 \times 6) + (4 \times 6)] \\ &= 2[32 + 48 + 24] \\ &= 2[104] \\ &= \underline{\underline{208 \text{ cm}^2}} \end{aligned}$$

✓

(c)



Solution:

Area of a cylinder $= 2\pi r(r + h)$

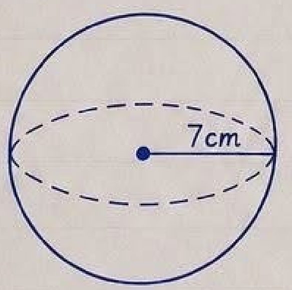
Where r = radius, h = height

Here, $r = 7\text{cm}$, $h = 10\text{cm}$ (use $\pi = \frac{22}{7}$)

$$\begin{aligned} \text{Area} &= 2 \times \frac{22}{7} \times 7(7 + 10) \\ &= 2 \times 2 \times 17 \\ &= \underline{\underline{748 \text{ cm}^2}} \end{aligned}$$

✓

(d)



Solution:

Area of a sphere $= 4\pi r^2$

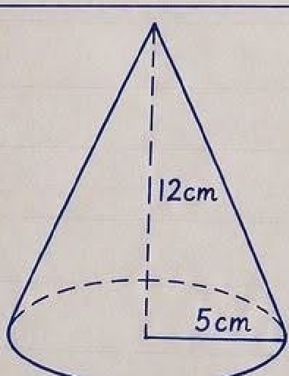
Where r = radius

Here, $r = 7\text{cm}$ (use $\pi = \frac{22}{7}$)

$$\begin{aligned} \text{Area} &= 4 \times \frac{22}{7} \times 7^2 \\ &= 4 \times \frac{22}{7} \times 49 \\ &= \underline{\underline{616 \text{ cm}^2}} \end{aligned}$$

✓

(e)



Solution:

Area of a cone $= \pi r(r + l)$

Where r = radius, l = slant height

First, find l using Pythagoras:

$l^2 = r^2 + h^2 = 5^2 + 12^2 = 25 + 144 = 169$

$l = \sqrt{169} = 13 \text{ cm}$

$$\begin{aligned} \text{Area} &= \frac{22}{7} \times 5(5 + 13) \\ &= \frac{110}{7} \times 18 \\ &= \frac{1980}{7} \\ &= \underline{\underline{282\tfrac{6}{7} \text{ cm}^2}} \end{aligned}$$

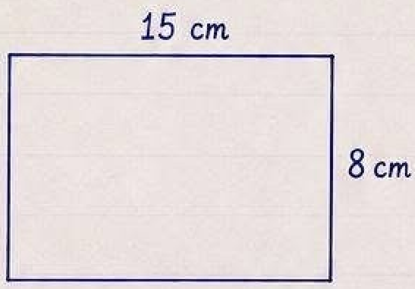
✓

Note: In all cases, the unit is cm^2.

✱ Find the perimeter of the following two-dimensional shapes.

PERIMETER OF A TWO-DIMENSIONAL SHAPE

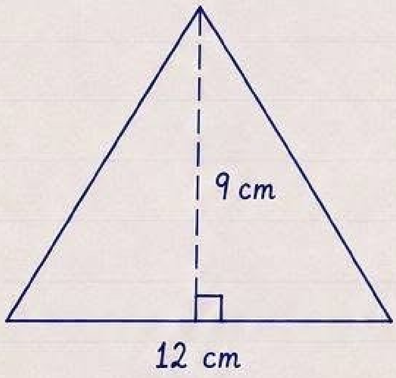
(a)



$$\text{Perimeter of rectangle} = 2(\ell + w)$$
$$= 2(\ell + w)$$
$$= 2(15 + 8)$$
$$= 2 \times 23$$
$$= \underline{\underline{46 \text{ cm}}}$$

✓

(b)



Perimeter of triangle $= a + b + c$

The triangle is isosceles.

Height $= 9$ cm, Base $= 12$ cm

Half of base $= \frac{12}{2} = 6$ cm

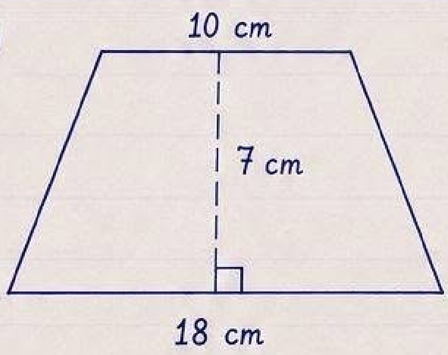
Each equal side $= \sqrt{9^2 + 6^2} = \sqrt{117} = 3\sqrt{13} \approx 10.82$ cm

Perimeter $= 12 + 10.82 + 10.82$

$= \underline{\underline{33.64 \text{ cm (approx.)}}}$

✓

(c)



Perimeter of trapezium $= a + b + c + d$

The trapezium is isosceles.

Difference of bases $= 18 - 10 = 8$ cm

Half of difference $= \frac{8}{2} = 4$ cm

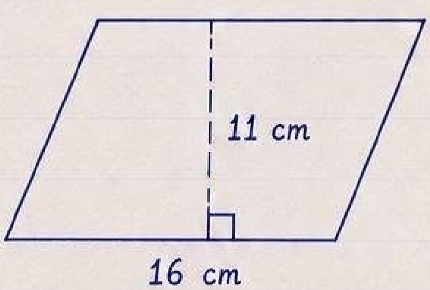
Each non-parallel side $= \sqrt{7^2 + 4^2} = \sqrt{65} \approx 8.06$ cm

Perimeter $= 10 + 18 + 8.06 + 8.06$

$= \underline{44.12 \text{ cm (approx.)}}$

✓

(d)



Perimeter of parallelogram $= 2(\text{base} + \text{side})$

(Note: 11 cm is the perpendicular height, not the side.)

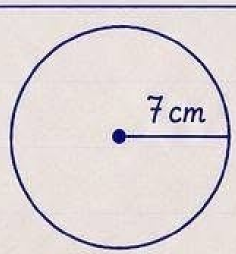
Let the side be a.

Perimeter $= 2(16 + a)$

$= 32 + 2a$ cm

(Numerical value cannot be found as the side is not given.)

(e)



$$\text{Perimeter of circle} = 2\pi r$$
$$= 2 \times \frac{22}{7} \times 7$$
$$= \underline{\underline{44 \text{ cm}}}$$

✓

Note : In all cases, the unit is cm.

→ Solve the following quadratic equation using factorization method.

Topic Box: QUADRATIC EQUATION BY FACTORIZATION METHOD.

(a) $x^2 + 5x + 6 = 0$

Solution:

$x^2 + 5x + 6 = 0$

$x^2 + 2x + 3x + 6 = 0$

$x(x+2) + 3(x+2) = 0$

$(x+2)(x+3) = 0$

$x + 2 = 0$ or $x + 3 = 0$

$x = -2$ $\qquad$ $x = -3$

(b) $x^2 - 7x + 12 = 0$

Solution:

$x^2 - 7x + 12 = 0$

$x^2 - 3x - 4x + 12 = 0$

$x(x-3) - 4(x-3) = 0$

$(x-3)(x-4) = 0$

$x - 3 = 0$ or $x - 4 = 0$

$x = 3$ $\qquad$ $x = 4$

(c) $x^2 + 2x - 15 = 0$

Solution:

$x^2 + 2x - 15 = 0$

$x^2 + 5x - 3x - 15 = 0$

$x(x+5) - 3(x+5) = 0$

$(x+5)(x-3) = 0$

$x + 5 = 0$ or $x - 3 = 0$

$x = -5$ $\qquad$ $x = 3$

(d) $2x^2 + x - 6 = 0$

Solution:

$2x^2 + x - 6 = 0$

$2x^2 + 4x - 3x - 6 = 0$

$2x(x+2) - 3(x+2) = 0$

$(2x-3)(x+2) = 0$

$2x - 3 = 0$ or $x + 2 = 0$

$2x = 3$ $\qquad$ $x = -2$

$x = \frac{3}{2}$ $\qquad$ $x = -2$

(e) $3x^2 - 5x - 2 = 0$

Solution:

$3x^2 - 5x - 2 = 0$

$3x^2 - 6x + x - 2 = 0$

$3x(x-2) + 1(x-2) = 0$

$(3x+1)(x-2) = 0$

$3x + 1 = 0$ or $x - 2 = 0$

$3x = -1$ $\qquad$ $x = 2$

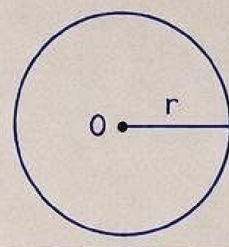
$x = -\frac{1}{3}$ $\qquad$ $x = 2$

AREAS RELATED TO CIRCLES

1 BASIC FORMULAS

- Let r = radius of the circle
- d = diameter = $2r$
- $\pi = \frac{22}{7}$ or 3.14

① Circumference $(C) = 2\pi r = \pi d$
② Area of circle $= \pi r^2$
③ Area of semicircle $= \frac{1}{2}\pi r^2$
④ Area of circle $= \frac{\pi d^2}{4}$



Area of circle $= \pi r^2$

Example :

If $r = 7$ cm, find the area of the circle.

$$\text{Area} = \pi r^2 = \frac{22}{7} \times 7^2$$
$$= \frac{22}{7} \times 49 = 154 \text{ cm}^2$$

2 AREA OF SECTOR

- θ = angle of sector (in degrees)
- Area of sector $= \frac{\theta}{360^\circ}\pi r^2$
- Arc length $(l) = \frac{\theta}{360^\circ} 2\pi r$

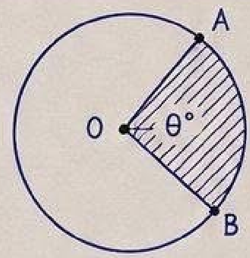
In terms of arc length (l) :
Area of sector $= \frac{1}{2} r l$

Example :

A sector of a circle has $\theta = 60^\circ$ and $r = 10$ cm.
Find (i) area of sector (ii) arc length.

(i) $\text{Area} = \frac{\theta}{360^\circ}\pi r^2 = \frac{60}{360^\circ} \times \pi \times 10^2$
$= \frac{1}{6} \times \pi \times 100 = \frac{50\pi}{3} \text{ cm}^2$

(ii) $\text{Arc length} = \frac{\theta}{360^\circ} 2\pi r = \frac{60}{360^\circ} \times 2 \times \pi \times 10$
$= \frac{10\pi}{3}$ cm



Area of sector $= \frac{\theta}{360^\circ}\pi r^2$
Arc length $= \frac{\theta}{360^\circ} 2\pi r$

3 AREA OF SEGMENT

- Area of segment = Area of sector − Area of triangle

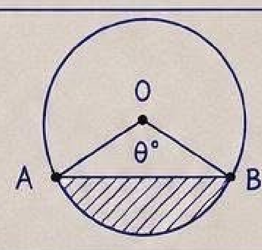
$$\boxed{\text{Area of segment} = \frac{\theta}{360^\circ}\pi r^2 - \frac{1}{2} r^2 \sin\theta}$$

(θ is the angle of the sector in degrees)

Example :

In a circle of radius 10 cm, a segment is cut off by a chord subtending an angle of 90° at the centre.
Find the area of the segment.

$$\text{Area of segment} = \frac{90}{360^\circ}\pi \times 10^2 - \frac{1}{2} \times 10^2 \times \sin 90^\circ$$
$$= \frac{1}{4} \times \pi \times 100 - \frac{1}{2} \times 100 \times 1$$
$$= 25\pi - 50 \text{ cm}^2 = (25\pi - 50) \text{ cm}^2$$



Area of segment $= \frac{\theta}{360^\circ}\pi r^2 - \frac{1}{2} r^2 \sin\theta$

(θ is in degrees)

4 QUADRANT & SEMICIRCLE

- Area of quadrant $= \frac{1}{4}\pi r^2$
- Area of semicircle $= \frac{1}{2}\pi r^2$

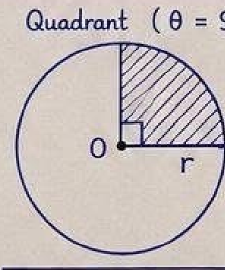
Example :

If $r = 14$ cm, find
(i) area of quadrant (ii) area of semicircle.

(i) Area of quadrant $= \frac{1}{4}\pi r^2 = \frac{1}{4} \times \pi \times 14^2$
$= \frac{1}{4} \times \pi \times 196 = 49\pi \text{ cm}^2$

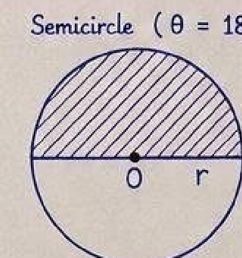
(ii) Area of semicircle $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 14^2$
$= \frac{1}{2} \times \pi \times 196 = 98\pi \text{ cm}^2$

Quadrant ($\theta = 90^\circ$)



Area of quadrant $= \frac{1}{4}\pi r^2$

Semicircle ($\theta = 180^\circ$)



Area of semicircle $= \frac{1}{2}\pi r^2$

TRIGONOMETRIC VALUES

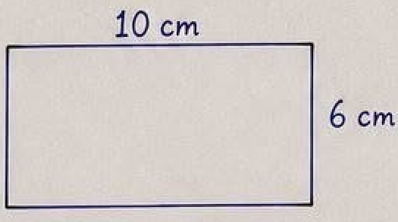
θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Undefined	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1
$\operatorname{cosec}\theta$	Undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Undefined

MATHEMATICS PRACTICE

1. Perimeter of a Rectangle

Question :

Find the perimeter of a rectangle whose length is 10 cm and breadth is 6 cm.



Solution :

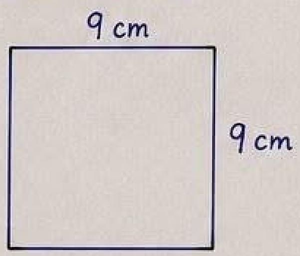
$$\begin{aligned}\text{Perimeter of a rectangle} &= 2\,(\ell + b)\\ &= 2\,(10 + 6)\\ &= 2 \times 16\\ &= 32 \text{ cm}\end{aligned}$$

Final Answer : 32 cm

2. Area of a Square

Question :

Find the area of a square whose side is 9 cm.



Solution :

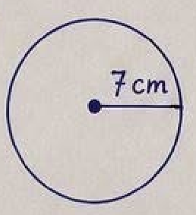
$$\begin{aligned}\text{Area of a square} &= \text{side} \times \text{side}\\ &= 9 \times 9\\ &= 81 \text{ cm}^2\end{aligned}$$

Final Answer : 81 cm^2

3. Circumference of a Circle

Question :

Find the circumference of a circle whose radius is 7 cm.
(Take $\pi = \frac{22}{7}$)



Solution :

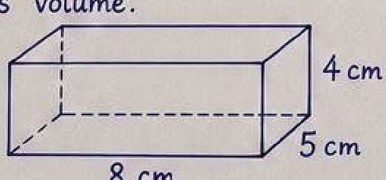
$$\begin{aligned}\text{Circumference} &= 2\pi r\\ &= 2 \times \frac{22}{7} \times 7\\ &= 44 \text{ cm}\end{aligned}$$

Final Answer : 44 cm

4. Volume of a Cuboid

Question :

A cuboid has length 8 cm, breadth 5 cm and height 4 cm. Find its volume.



Solution :

$$\begin{aligned}\text{Volume of a cuboid} &= \ell \times b \times h\\ &= 8 \times 5 \times 4\\ &= 160 \text{ cm}^3\end{aligned}$$

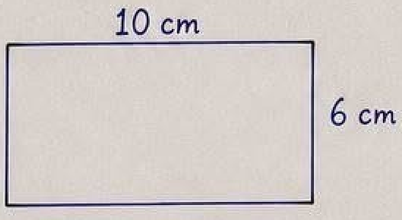
Final Answer : 160 cm^3

MATHEMATICS PRACTICE

1. Perimeter of a Rectangle

Question :

Find the perimeter of a rectangle whose length is 10 cm and breadth is 6 cm.



Solution :

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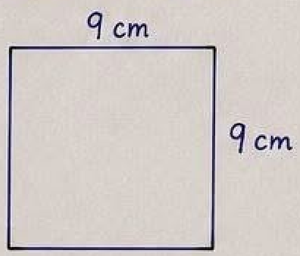


Solution :

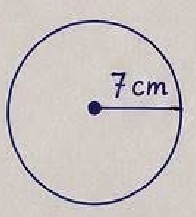
$$\begin{aligned}\text{Area of a square} &= \text{side} \times \text{side} \\ &= 9 \times 9 \\ &= 81 \text{ cm}^2\end{aligned}$$

Final Answer : 81 cm^2

3. Circumference of a Circle

Question :

Find the circumference of a circle whose radius is 7 cm. (Take $\pi = \frac{22}{7}$)



Solution :

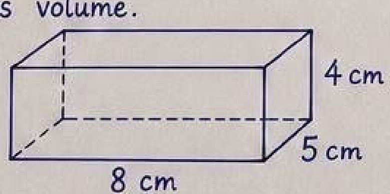
$$\begin{aligned}\text{Circumference} &= 2\pi r \\ &= 2 \times \frac{22}{7} \times 7 \\ &= 44 \text{ cm}\end{aligned}$$

Final Answer : 44 cm

4. Volume of a Cuboid

Question :

A cuboid has length 8 cm, breadth 5 cm and height 4 cm. Find its volume.



Solution :

$$\begin{aligned}\text{Volume of a cuboid} &= \ell \times b \times h \\ &= 8 \times 5 \times 4 \\ &= 160 \text{ cm}^3\end{aligned}$$

Final Answer : 160 cm^3

LCM AND HCF OF A NUMBER

Find the LCM and HCF of the following

(a) 12 and 18

Solution:

$12 = 2 \times 2 \times 3$

$18 = 2 \times 3 \times 3$

$HCF = 2 \times 3 = \underline{\underline{6}}$

$LCM = 2 \times 2 \times 3 \times 3 = \underline{\underline{36}}$

(b) 15 and 20

Solution:

$15 = 3 \times 5$

$20 = 2 \times 2 \times 5$

$HCF = \underline{\underline{5}}$

$LCM = 2 \times 2 \times 3 \times 5 = \underline{\underline{60}}$

(c) 24 and 36

Solution:

$24 = 2 \times 2 \times 2 \times 3 \quad (= 2^3 \times 3)$

$36 = 2 \times 2 \times 3 \times 3 \quad (= 2^2 \times 3^2)$

$HCF = 2^2 \times 3 = 4 \times 3 = \underline{\underline{12}}$

$LCM = 2^3 \times 3^2 = 8 \times 9 = \underline{\underline{72}}$

(d) 14 and 21

Solution:

$14 = 2 \times 7$

$21 = 3 \times 7$

$HCF = \underline{\underline{7}}$

$LCM = 2 \times 3 \times 7 = \underline{\underline{42}}$

(e) 16 and 24

Solution:

$16 = 2 \times 2 \times 2 \times 2 \quad (= 2^4)$

$24 = 2 \times 2 \times 2 \times 3 \quad (= 2^3 \times 3)$

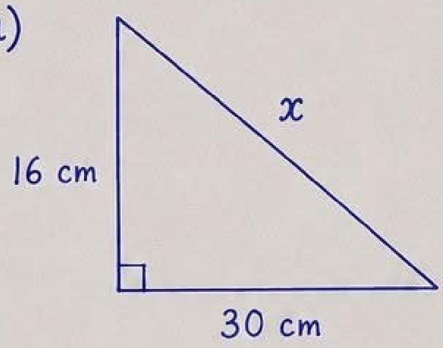
$HCF = 2^3 = \underline{\underline{8}}$

$LCM = 2^4 \times 3 = 16 \times 3 = \underline{\underline{48}}$

→ Find the unknown side lengths of each of the following figures

USING PYTHAGORAS THEOREM

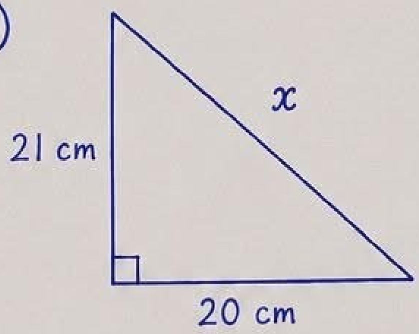
(a)



$$x^2 = 16^2 + 30^2$$
$$x^2 = 256 + 900$$
$$x^2 = 1156$$
$$x = \sqrt{1156}$$
$$x = \underline{\underline{34 \text{ cm}}}$$

✓

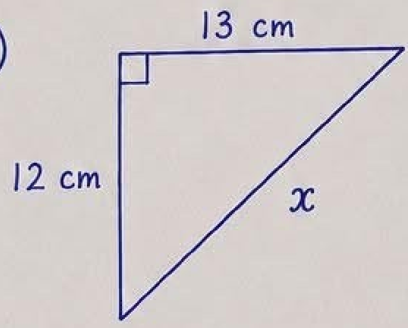
(b)



$$x^2 = 21^2 + 20^2$$
$$x^2 = 441 + 400$$
$$x^2 = 841$$
$$x = \sqrt{841}$$
$$x = \underline{\underline{29 \text{ cm}}}$$

✓

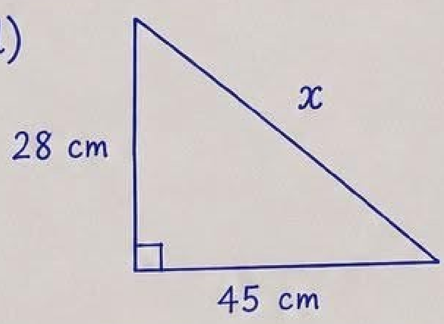
(c)



$$x^2 = 12^2 + 13^2$$
$$x^2 = 144 + 169$$
$$x^2 = 313$$
$$x = \sqrt{313}$$
$$x = \underline{\underline{17.69 \text{ cm}}}$$

✓

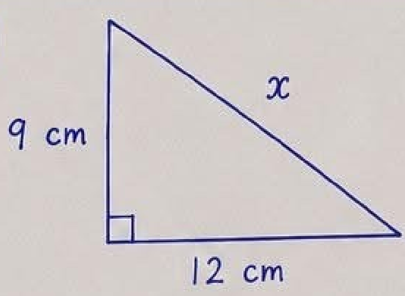
(d)



$$x^2 = 28^2 + 45^2$$
$$x^2 = 784 + 2025$$
$$x^2 = 2809$$
$$x = \sqrt{2809}$$
$$x = \underline{\underline{53 \text{ cm}}}$$

✓

(e)



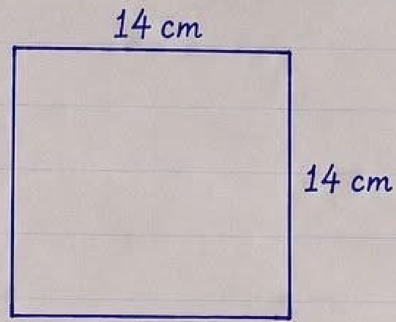
$$x^2 = 9^2 + 12^2$$
$$x^2 = 81 + 144$$
$$x^2 = 225$$
$$x = \sqrt{225}$$
$$x = \underline{\underline{15 \text{ cm}}}$$

✓

* Find the area of the following two-dimensional shapes.

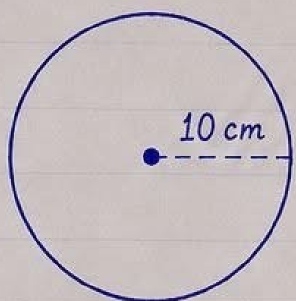
Area of Common 2D Shapes

(a)



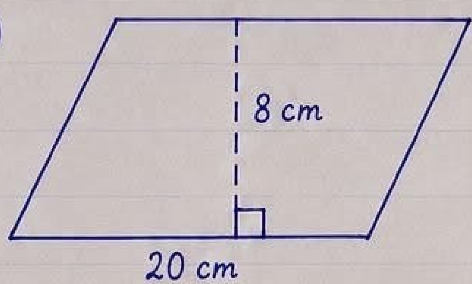
$$\text{Area of square} = \text{side} \times \text{side}$$
$$= a \times a$$
$$= 14 \times 14$$
$$= \underline{\underline{196 \text{ cm}^2}} \checkmark$$

(b)



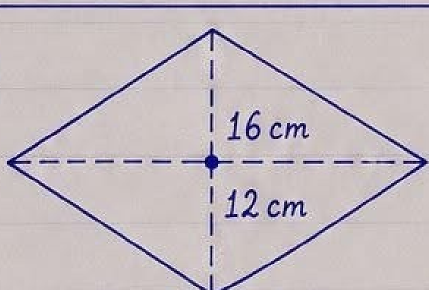
$$\text{Area of circle} = \pi r^2$$
$$= \frac{22}{7} \times 10 \times 10$$
$$= \frac{2200}{7}$$
$$= \underline{\underline{314.29 \text{ cm}^2}} \checkmark$$

(c)



$$\text{Area of parallelogram} = \text{base} \times \text{height}$$
$$= b \times h$$
$$= 20 \times 8$$
$$= \underline{\underline{160 \text{ cm}^2}} \checkmark$$

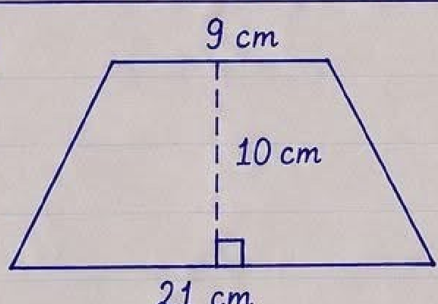
(d)



$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$
$$= \frac{1}{2} \times 16 \times 12$$
$$= \underline{\underline{96 \text{ cm}^2}} \checkmark$$

(Here, 16 cm and 12 cm are the full diagonals)

(e)

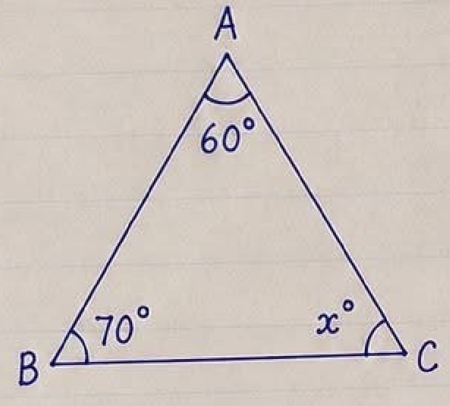


$$\text{Area of trapezium} = \frac{1}{2} \times (a+b) \times h$$
$$= \frac{1}{2} \times (9 + 21) \times 10$$
$$= \frac{1}{2} \times 30 \times 10$$
$$= \underline{\underline{150 \text{ cm}^2}} \checkmark$$

Note: In all cases, the unit is cm^2.

Solve for X in the diagram below.

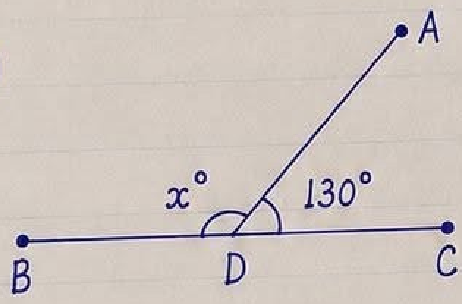
(1)



In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ \text{ (Sum of angles in a triangle)}$$
$$60^\circ + 70^\circ + x = 180^\circ$$
$$130^\circ + x = 180^\circ$$
$$x = 180^\circ - 130^\circ$$
$$x = \underline{\underline{50^\circ}}$$

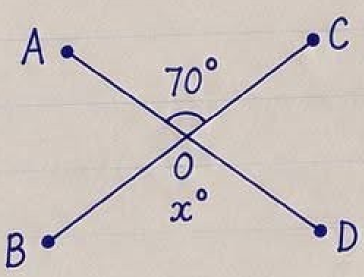
(2)



Since BDC is a straight line,

$$\angle BDA + \angle ADC = 180^\circ \text{ (angles on a straight line)}$$
$$x + 130^\circ = 180^\circ$$
$$x = 180^\circ - 130^\circ$$
$$x = \underline{\underline{50^\circ}}$$

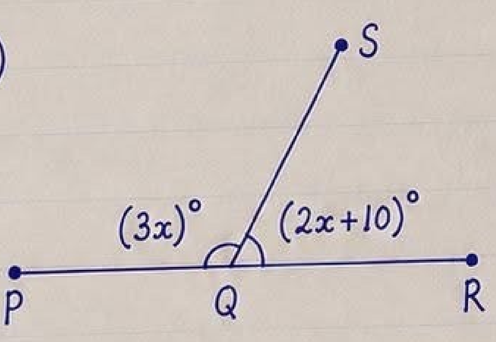
(3)



Vertical opposite angles are equal.

$$\angle AOC = \angle BOD$$
$$x = \underline{\underline{70^\circ}}$$

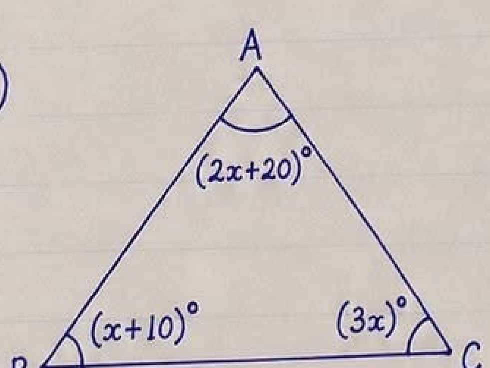
(4)



Since PQR is a straight line,

$$(3x)^\circ + (2x+10)^\circ = 180^\circ$$
$$3x + 2x + 10 = 180^\circ$$
$$5x + 10 = 180^\circ$$
$$5x = 180^\circ - 10^\circ$$
$$5x = 170^\circ$$
$$x = \frac{170^\circ}{5}$$
$$x = \underline{\underline{34^\circ}}$$

(5)



In $\triangle ABC$,

$$(2x+20)^\circ + (x+10)^\circ + (3x)^\circ = 180^\circ$$
$$2x + 20 + x + 10 + 3x = 180^\circ$$
$$6x + 30 = 180^\circ$$
$$6x = 150^\circ$$
$$x = \frac{150^\circ}{6}$$
$$x = \underline{\underline{25^\circ}}$$

Solve the following quadratic equation using completing the square method.

QUADRATIC EQUATION BY COMPLETING THE SQUARE METHOD.

(a) $x^2 + 6x + 5 = 0$

Solution:
$$
\begin{aligned}
& x^2 + 6x + 5 = 0 \\
\Rightarrow\ & x^2 + 6x = -5 \\
\Rightarrow\ & x^2 + 6x + 9 = -5 + 9 \\
\Rightarrow\ & (x + 3)^2 = 4 \\
\Rightarrow\ & x + 3 = \pm 2 \\
\Rightarrow\ & x = -3 \pm 2 \\
\Rightarrow\ & \underline{\underline{x = -1, -5}}
\end{aligned}
$$

✓

(b) $x^2 - 4x - 12 = 0$

Solution:
$$
\begin{aligned}
& x^2 - 4x - 12 = 0 \\
\Rightarrow\ & x^2 - 4x = 12 \\
\Rightarrow\ & x^2 - 4x + 4 = 12 + 4 \\
\Rightarrow\ & (x - 2)^2 = 16 \\
\Rightarrow\ & x - 2 = \pm 4 \\
\Rightarrow\ & x = 2 \pm 4 \\
\Rightarrow\ & \underline{\underline{x = -2, 6}}
\end{aligned}
$$

✓

(c) $x^2 + 2x - 3 = 0$

Solution:
$$
\begin{aligned}
& x^2 + 2x - 3 = 0 \\
\Rightarrow\ & x^2 + 2x = 3 \\
\Rightarrow\ & x^2 + 2x + 1 = 3 + 1 \\
\Rightarrow\ & (x + 1)^2 = 4 \\
\Rightarrow\ & x + 1 = \pm 2 \\
\Rightarrow\ & x = -1 \pm 2 \\
\Rightarrow\ & \underline{\underline{x = -3, 1}}
\end{aligned}
$$

✓

(d) $2x^2 + 8x - 6 = 0$

Solution:
$$
\begin{aligned}
& 2x^2 + 8x - 6 = 0 \\
\Rightarrow\ & 2(x^2 + 4x) = 6 \\
\Rightarrow\ & x^2 + 4x = 3 \\
\Rightarrow\ & x^2 + 4x + 4 = 3 + 4 \\
\Rightarrow\ & (x + 2)^2 = 7 \\
\Rightarrow\ & x + 2 = \pm \sqrt{7} \\
\Rightarrow\ & \underline{\underline{x = -2 \pm \sqrt{7}}}
\end{aligned}
$$

✓

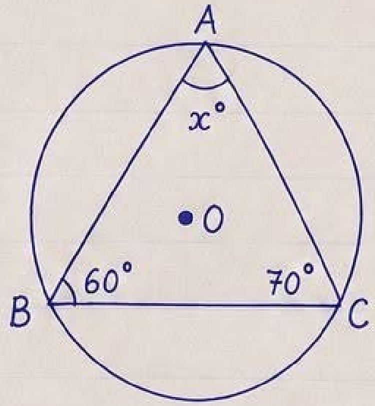
(e) $3x^2 - 6x - 9 = 0$

Solution:
$$
\begin{aligned}
& 3x^2 - 6x - 9 = 0 \\
\Rightarrow\ & 3(x^2 - 2x) = 9 \\
\Rightarrow\ & x^2 - 2x = 3 \\
\Rightarrow\ & x^2 - 2x + 1 = 3 + 1 \\
\Rightarrow\ & (x - 1)^2 = 4 \\
\Rightarrow\ & x - 1 = \pm 2 \\
\Rightarrow\ & x = 1 \pm 2 \\
\Rightarrow\ & \underline{\underline{x = -1, 3}}
\end{aligned}
$$

✓

Solve for X in the diagram below.

(1.)



In $\triangle ABC$,

$\angle A + \angle B + \angle C = 180°$ (Sum of angles in a triangle)

$60° + 70° + x = 180°$

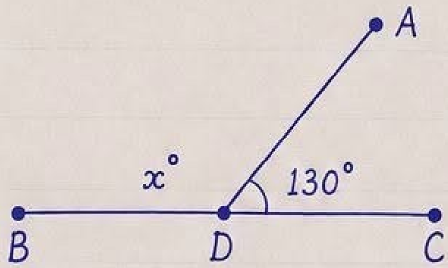
$130° + x = 180°$

$x = 180° - 130°$

$x = 50°$

$\underline{\underline{x = 50°}}$ ✓

(2.)



Since BDC is a straight line,

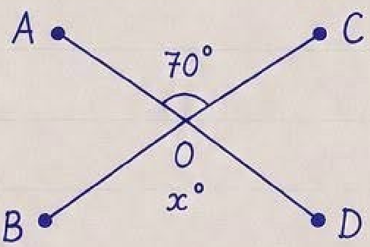
$\angle BDA + \angle ADC = 180°$ (angles on a straight line)

$x + 130° = 180°$

$x = 180° - 130°$

$\underline{\underline{x = 50°}}$ ✓

(3.)

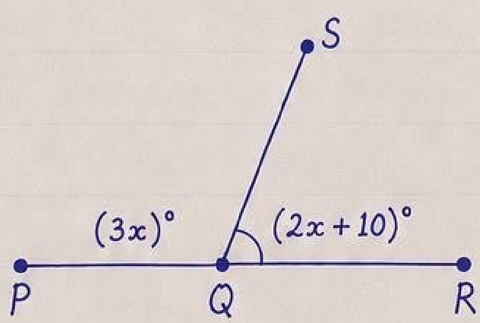


Vertical opposite angles are equal.

$\angle AOC = \angle BOD$

$x = \underline{\underline{70°}}$ ✓

(4.)



Since PQR is a straight line,

$(3x)° + (2x+10)° = 180°$

$3x + 2x + 10 = 180°$

$5x + 10 = 180°$

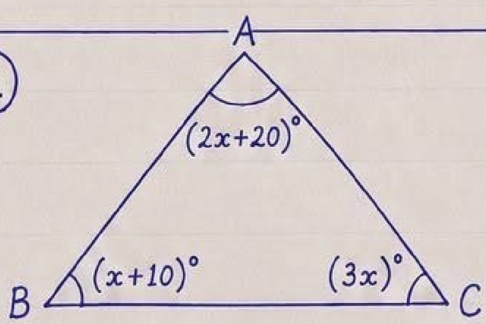
$5x = 180° - 10°$

$5x = 170°$

$x = \frac{170°}{5}$

$\underline{\underline{x = 34°}}$ ✓

(5.)



In $\triangle ABC$,

$(2x + 20)° + (x + 10)° + (3x)° = 180°$

$2x + 20 + x + 10 + 3x = 180°$

$6x + 30 = 180°$

$6x = 150°$

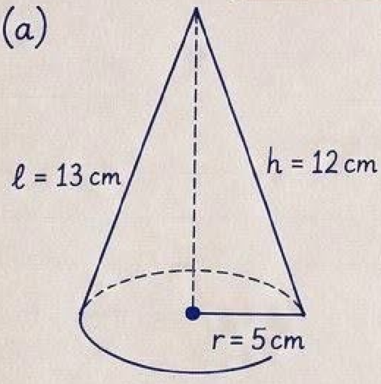
$x = \frac{150°}{6}$

$\underline{\underline{x = 25°}}$ ✓

✳ Find the perimeter / circumference of the following shapes.

TOPIC BOX : PERIMETER / CIRCUMFERENCE OF SHAPES

(a)



Perimeter (total) around the base of a cone

= circumference of base

$= 2\pi r$

$= 2 \times \pi \times 5$

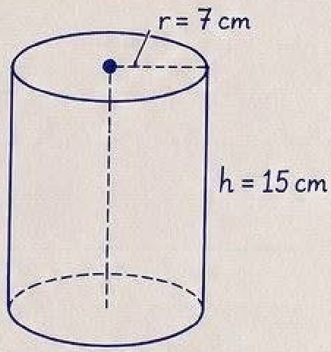
$= 10\pi$ cm

$= 10 \times 3.142$

= <u>31.42 cm</u>

> Formula :
> Circumference of base $= 2\pi r$
> ($\pi = 3.142$)

(b)



Perimeter (total) around the base of a cylinder

= circumference of base

$= 2\pi r$

$= 2 \times \pi \times 7$

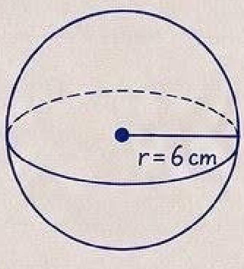
$= 14\pi$ cm

$= 14 \times 3.142$

= <u>43.99 cm</u>

> Formula :
> Circumference of base $= 2\pi r$
> ($\pi = 3.142$)

(c)



Perimeter is not defined for a sphere.
However, the circumference of a great circle of a sphere $= 2\pi r$

$= 2 \times \pi \times 6$

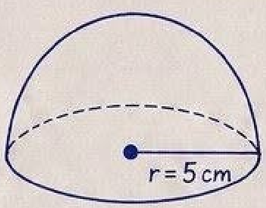
$= 12\pi$ cm

$= 12 \times 3.142$

= <u>37.70 cm</u>

> Formula :
> Circumference of great circle $= 2\pi r$
> ($\pi = 3.142$)

(d)



Perimeter (total) around the base (rim) of a hemisphere

= circumference of base

$= 2\pi r$

$= 2 \times \pi \times 5$

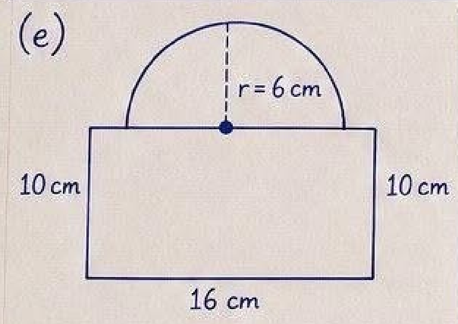
$= 10\pi$ cm

$= 10 \times 3.142$

= <u>31.42 cm</u>

> Formula :
> Circumference of base $= 2\pi r$
> ($\pi = 3.142$)

(e)



Perimeter of the composite shape = sum of outer boundaries

= base (16) + two vertical sides (10 + 10) + semicircle arc (πr)

$= 16 + 10 + 10 + \pi \times 6$

$= 36 + 6\pi$

$= 36 + 6 \times 3.142$

$= 36 + 18.852$

= <u>54.85 cm</u>

> Note :
> Only the outer boundary is added.
> Semicircle arc $= \pi r$

IMPORTANT REMINDERS

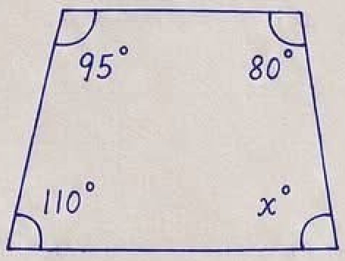
- Perimeter is defined only for 2D shapes.
- Perimeter means the total distance around the outer boundary.
- Use $\pi = 3.142$ unless otherwise stated.
- Write your answer with the correct unit (cm).

You can do it!
Practice makes perfect. ♡

⋆ Find the unknown angle of the following quadrilateral.

Topic Box : UNKNOWN ANGLE OF A QUADRILATERAL

(a)



Sum of angles in a quadrilateral = 360°

$95^\circ + 80^\circ + 110^\circ + x^\circ = 360^\circ$

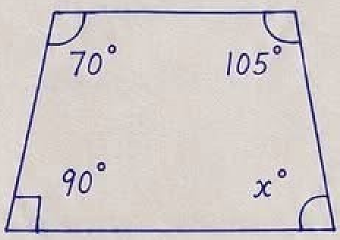
$285^\circ + x^\circ = 360^\circ$

$x^\circ = 360^\circ - 285^\circ$

$x^\circ = \underline{\underline{75^\circ}}$

✓

(b)



Sum of angles in a quadrilateral = 360°

$70^\circ + 105^\circ + 90^\circ + x^\circ = 360^\circ$

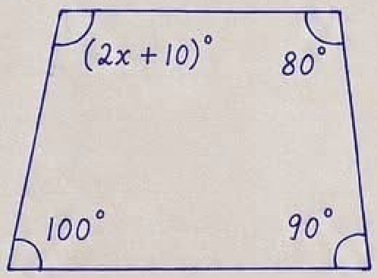
$265^\circ + x^\circ = 360^\circ$

$x^\circ = 360^\circ - 265^\circ$

$x^\circ = \underline{\underline{95^\circ}}$

✓

(c)



Sum of angles in a quadrilateral = 360°

$(2x + 10)^\circ + 80^\circ + 90^\circ + 100^\circ = 360^\circ$

$2x + 10 + 80 + 90 + 100 = 360$

$2x + 280 = 360$

$2x = 360 - 280$

$2x = 80$

$x = \underline{\underline{40^\circ}}$

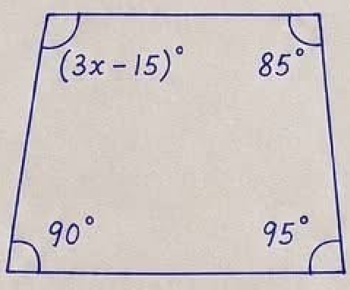
Check :

$2(40) + 10 = 80 + 10 = 90^\circ$

Angles are $90^\circ, 80^\circ, 90^\circ, 100^\circ$; $90 + 80 + 90 + 100 = 360^\circ$.

✓

(d)



Sum of angles in a quadrilateral = 360°

$(3x - 15)^\circ + 85^\circ + 95^\circ + 90^\circ = 360^\circ$

$3x - 15 + 85 + 95 + 90 = 360$

$3x + 255 = 360$

$3x = 360 - 255$

$3x = 105$

$x = \underline{\underline{35^\circ}}$

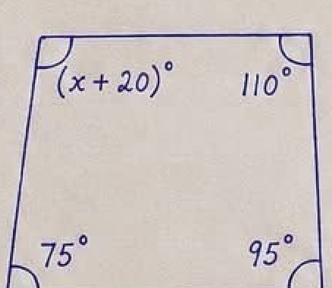
Check :

$3(35) - 15 = 105 - 15 = 90^\circ$

Angles are $90^\circ, 85^\circ, 95^\circ, 90^\circ$; Sum = 360°.

✓

(e)



Sum of angles in a quadrilateral = 360°

$(x + 20)^\circ + 110^\circ + 95^\circ + 75^\circ = 360^\circ$

$x + 20 + 110 + 95 + 75 = 360$

$x + 300 = 360$

$x = 360 - 300$

$x = \underline{\underline{60^\circ}}$

Check :

$x + 20 = 60 + 20 = 80^\circ$

Angles are $80^\circ, 110^\circ, 95^\circ, 75^\circ$; $80 + 110 + 95 + 75 = 360^\circ$.

✓

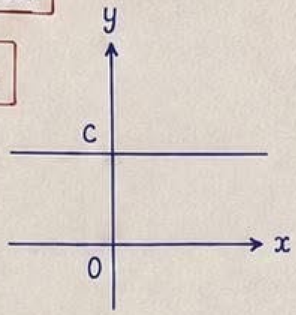
FAMOUS FUNCTIONS & THEIR GRAPHS

1 CONSTANT FUNCTION

$y = c, \quad c \in \mathbb{R}$

Domain : $\mathbb{R}$

Range : $\{c\}$

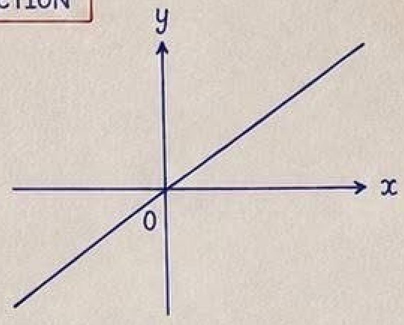


2 IDENTITY FUNCTION

$y = x$

Domain : $\mathbb{R}$

Range : $\mathbb{R}$



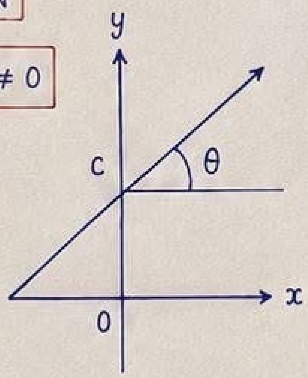
3 LINEAR FUNCTION

$y = mx + c, \quad m \neq 0$

Domain : $\mathbb{R}$

Range : $\mathbb{R}$

Slope $(m) = \tan\theta$



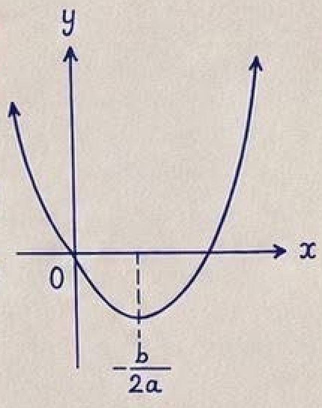
4 QUADRATIC FUNCTION

$y = ax^2 + bx + c, \quad a \neq 0$

Domain : $\mathbb{R}$

Range : $\begin{cases} \left[f\left(-\frac{b}{2a}\right), \infty\right) & \text{if } a>0 \\ \left(-\infty, f\left(-\frac{b}{2a}\right)\right] & \text{if } a<0 \end{cases}$

where $f\left(-\frac{b}{2a}\right) = \frac{4ac-b^2}{4a}$



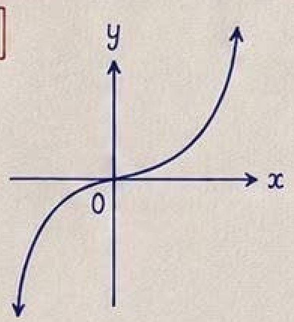
5 CUBIC FUNCTION

$y = x^3$

Domain : $\mathbb{R}$

Range : $\mathbb{R}$

(Typical cubic graph)

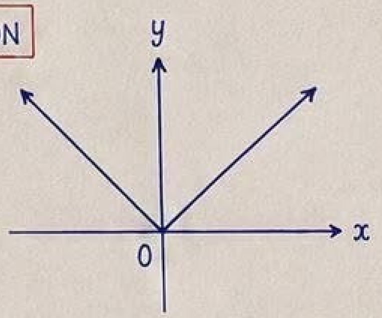


6 MODULUS FUNCTION

$y = |x|$

Domain : $\mathbb{R}$

Range : $[0, \infty)$

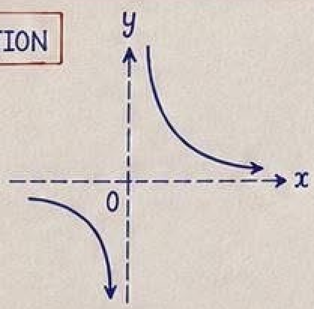


7 RECIPROCAL FUNCTION

$y = \frac{1}{x}$

Domain : $\mathbb{R} - \{0\}$

Range : $\mathbb{R} - \{0\}$

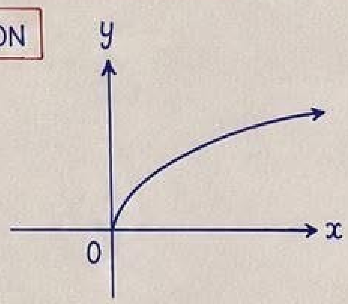


8 SQUARE ROOT FUNCTION

$y = \sqrt{x}$

Domain : $[0, \infty)$

Range : $[0, \infty)$



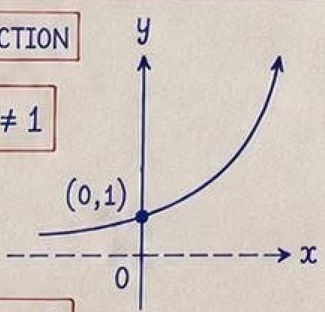
9 EXPONENTIAL FUNCTION

$y = a^x, \quad a>0, \quad a \neq 1$

Domain : $\mathbb{R}$

Range : $(0, \infty)$

Horizontal asymptote : $y = 0$



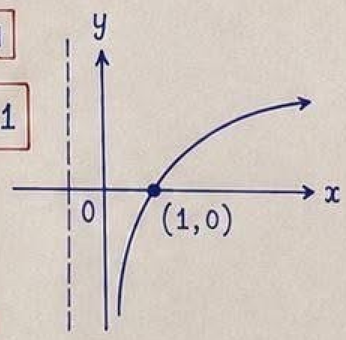
10 LOGARITHMIC FUNCTION

$y = \log_a x, \quad a>0, \quad a \neq 1$

Domain : $(0, \infty)$

Range : $\mathbb{R}$

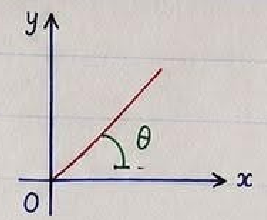
Vertical asymptote : $x = 0$



STRAIGHT LINE

1. Slope (m)

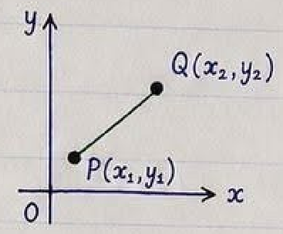
Slope of a line making angle θ with the positive x-axis.



$$\boxed{m = \tan\theta}$$

2. Slope Between Two Points

For two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ $(x_1 \neq x_2)$.



$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$(x_1 \neq x_2)$

3. Slope – Intercept Form

Line with slope m and y-intercept c.

$$\boxed{y = mx + c}$$

$(c = y\text{-intercept})$

4. Point – Slope Form

Line with slope m passing through point (x_1, y_1).

$$\boxed{y - y_1 = m\,(x - x_1)}$$

5. Two – Point Form

Line passing through two points (x_1, y_1) and (x_2, y_2) $(x_1 \neq x_2)$.

$$\boxed{\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}}$$

6. Intercept Form

Line cuts the axes at $A(a, 0)$ and $B(0, b)$. $(a \neq 0,\ b \neq 0)$

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

7. General Form

The most general equation of a straight line.

$$\boxed{Ax + By + C = 0}$$

where A, B not both zero. $(A^2 + B^2 \neq 0)$

8. Distance of Point From Line

Distance of point (x_1, y_1) from line $Ax + By + C = 0$

$$\boxed{d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}}$$

9. Angle Between Two Lines

If lines have slopes m_1 and m_2, then angle θ between them is

$$\boxed{\tan\theta = \frac{|m_1 - m_2|}{1 + m_1 m_2}}$$

$0 \leq \theta < 90^\circ$

10. Conditions For Parallel and Perpendicular Lines

- Lines are parallel $\Leftrightarrow$ their slopes are equal. $\boxed{m_1 = m_2}$
- Lines are perpendicular $\Leftrightarrow$ product of their slopes is -1. $\boxed{A_1A_2 + B_1B_2 = 0}$

(for lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$)