MATHEMATICS FORMULA SHEET

1. ARITHMETIC FORMULAS

- Sum of first n natural numbers: $\boxed{S_n = \frac{n(n+1)}{2}}$
- Sum of squares of first n natural numbers: $\boxed{S_n = \frac{n(n+1)(2n+1)}{6}}$
- Sum of cubes of first n natural numbers: $\boxed{S_n = \left[\frac{n(n+1)}{2}\right]^2}$

2. ALGEBRA FORMULAS

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a+b)(a-b)$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Example:

If $a = 3$, $b = 2$, find $(a+b)^2$

Solution:

$$
\begin{aligned}
(a+b)^2 &= a^2 + 2ab + b^2 \\
&= 3^2 + 2(3)(2) + 2^2 \\
&= 9 + 12 + 4 \\
&= 25
\end{aligned}
$$

3. QUADRATIC EQUATION

Standard form: $ax^2 + bx + c = 0 \quad (a \neq 0)$

Roots: $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

Nature of roots:

- $b^2 - 4ac > 0$: Real and distinct
- $b^2 - 4ac = 0$: Real and equal
- $b^2 - 4ac < 0$: Imaginary

Example:

Solve $2x^2 - 5x + 3 = 0$

Here, $a = 2$, $b = -5$, $c = 3$

$$x = \frac{5 \pm \sqrt{25 - 24}}{4} = \frac{5 \pm 1}{4}$$

$$x = \frac{6}{4}, \frac{4}{4} = \frac{3}{2}, 1$$

4. COORDINATE GEOMETRY

- Distance between (x_1, y_1) and (x_2, y_2):

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

- Midpoint of the line joining (x_1, y_1) and (x_2, y_2):

$$\boxed{M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

Example: Find distance between $A(1,2)$ and $B(4,6)$

$$d = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{3^3 + 4^2}$$

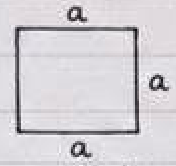
$$= \sqrt{9 + 16} = \sqrt{25} = 5$$

☆ Practice Makes Perfect! ☆

5. GEOMETRY FORMULAS

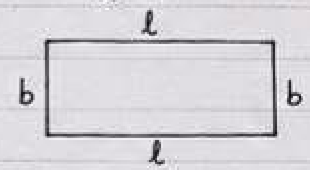
- Square:

  Area $= a^2$, Perimeter $= 4a$



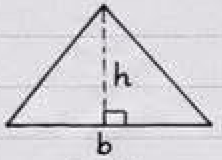
- Rectangle:

  Area $= l \times b$, Perimeter $= 2(l+b)$



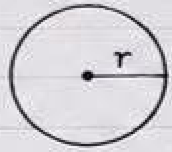
- Triangle:

  Area $= \frac{1}{2} bh$



- Circle:

  Area $= \pi r^2$, Circumference $= 2\pi r$



Example:

Find area of a circle of radius 7 cm.

Solution:

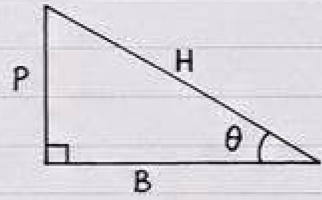
Area $= \pi r^2 = \frac{22}{7} \times 7^2 = 154 \text{ cm}^2$

$\left(\text{Take } \pi = \frac{22}{7}\right)$

6. TRIGONOMETRY FORMULAS

- Basic Ratios:

  $\sin\theta = \frac{P}{H}$, $\cos\theta = \frac{B}{H}$, $\tan\theta = \frac{P}{B}$



where P = Perpendicular, B = Base, H = Hypotenuse

- Identities:

  (i) $\sin^2\theta + \cos^2\theta = 1$

  (ii) $1 + \tan^2\theta = \sec^2\theta$

  (iii) $1 + \cot^2\theta = \text{cosec}^2\theta$

Example:

If $\sin\theta = \frac{3}{5}$, find $\cos\theta$

Solution:

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$

$\cos\theta = \frac{4}{5}$ (Taking +ve value)

7. MENSURATION FORMULAS

- Cube:

  Volume $= a^3$, TSA $= 6a^2$

- Cuboid:

  Volume $= lbh$, TSA $= 2(lb + bh + hl)$

- Cylinder:

  Volume $= \pi r^2 h$, CSA $= 2\pi rh$, TSA $= 2\pi r(r+h)$

Example:

Find volume of a cuboid whose $l = 5$ cm, $b = 4$ cm, and $h = 3$ cm.

Solution:

Volume $= lbh$

$= 5 \times 4 \times 3$

$= 60 \text{ cm}^3$

Learn Formulas Well, Solve Problems Even Better! ☺

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8. STATISTICS FORMULAS

- Mean (Average): $\bar{x} = \dfrac{\text{Sum of all observations}}{\text{Total number of observations}} = \dfrac{\Sigma x_i}{n}$
- Median: Middle value of arranged data.
- Mode: Value that occurs most frequently.

Range: R = Largest value − Smallest value

- Variance (σ^2): $\sigma^2 = \dfrac{\Sigma (x_i - \bar{x})^2}{n}$ (population)
- Standard Deviation (σ): $\sigma = \sqrt{\dfrac{\Sigma (x_i - \bar{x})^2}{n}}$

Example:
Find mean of 2, 4, 6, 8, 10

Solution:

$\bar{x} = \dfrac{2+4+6+8+10}{5}$

$= \dfrac{30}{5} = 6$

9. PROBABILITY FORMULAS

- Probability of an event E:

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$0 \le P(E) \le 1$

- Complementary Event:

$P(E') = 1 - P(E)$

- Probability of union (Mutually exclusive events):

$P(A \cup B) = P(A) + P(B)$

- Probability of intersection (Independent events):

$P(A \cap B) = P(A) \times P(B)$

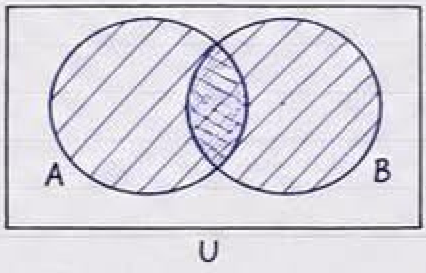
Example:
A dice is thrown.
Find P("3").

Solution:
Total outcomes = 6
Favourable outcome = 1

$P(3) = \dfrac{1}{6}$

10. SETS FORMULAS

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A') = n(U) - n(A)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
$- n(A \cap B) - n(B \cap C) - n(C \cap A)$
$+ n(A \cap B \cap C)$



11. EXPONENTS & RADICALS

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ $(b \neq 0)$
- $a^0 = 1$ $(a \neq 0)$
- $a^{-n} = \dfrac{1}{a^n}$
- $\sqrt[n]{a} = a^{1/n}$

Example:
Simplify: $\dfrac{(2^3 \times 2^5)}{2^4}$

Solution:

$= \dfrac{2^{3+5}}{2^4} = 2^{8-4} = 2^4$

$= 16$

☆ Practice More, Score More! ☆

12. FACTORIZATION FORMULAS

- $a^2 - b^2 = (a-b)(a+b)$
- $a^2 + 2ab + b^2 = (a+b)^2$
- $a^2 - 2ab + b^2 = (a-b)^2$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

Example:

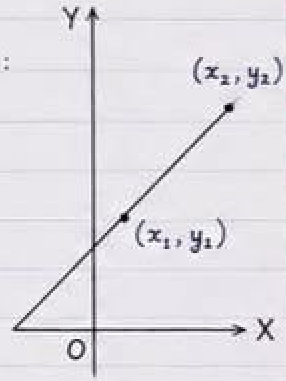
Factorize: $x^2 - 5x + 6$

Solution:

$$\begin{aligned} x^2 - 5x + 6 &= x^2 - 2x - 3x + 6 \\ &= x(x-2) - 3(x-2) \\ &= (x-2)(x-3) \end{aligned}$$

13. LINEAR EQUATIONS

- General form (two variables): $ax + by + c = 0$
- Slope (m) of line: $m = -\frac{a}{b}$ $(b \neq 0)$
- Equation of line through (x_1, y_1) with slope m:
  $y - y_1 = m(x - x_1)$
- Equation of line through two points (x_1, y_1) and (x_2, y_2):
  $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- Distance between two points:
  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Example:

Find slope of $2x - 3y + 6 = 0$

Solution:

$$\begin{aligned} 2x - 3y + 6 &= 0 \\ \Rightarrow -3y &= -2x - 6 \\ \Rightarrow y &= \frac{2}{3}x + 2 \end{aligned}$$

Slope, $m = \frac{2}{3}$

14. POLYNOMIALS

- Degree of a polynomial: highest power of the variable.
- If $p(x)$ and $g(x)$ are polynomials then
  (i) $(p+g)(x) = p(x) + g(x)$
  (ii) $(p-g)(x) = p(x) - g(x)$
  (iii) $(p \times g)(x) = p(x) \times g(x)$

Example:

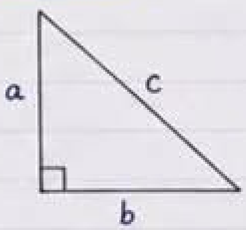
If $p(x) = 2x^2 + 3x - 1$
and $g(x) = x^2 - x + 2$
Find $(p+g)(x)$.

Solution:

$$\begin{aligned} (p+g)(x) &= p(x) + g(x) \\ &= (2x^2 + 3x - 1) + (x^2 - x + 2) \\ &= 3x^2 + 2x + 1 \end{aligned}$$

15. MISCELLANEOUS IMPORTANT FORMULAS

- Circumference of circle: $2\pi r$
- Area of circle: πr^2
- Area of triangle: $\frac{1}{2}bh$
- Pythagoras Theorem: $a^2 + b^2 = c^2$
  (In right-angled $\triangle$)



- Surface area of cube: $6a^2$
- Volume of cube: a^3
- Surface area of cuboid:
  $2(lb + bh + hl)$
- Volume of cuboid: $l \times b \times h$

Good Formulas + Regular Practice = Mathematics Success! ☺

MATHEMATICS FORMULA SHEET

16. RATIO & PROPORTION

- If $a : b = c : d$, then $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$
- Componendo: $a + b : b = c + d : d$
- Dividendo: $a - b : b = c - d : d$ $(b, d \neq 0)$
- Inverse ratio: $b : a$
- Continued proportion: If $a : b = b : c$, then a, b, c are in continued proportion.

Example:

If $2 : 3 = 4 : 6$, find

(i) $(2+3) : 3$

(ii) $(2-3) : 3$

Solution:

(i) $5 : 3$

(ii) $-1 : 3$

17. PERCENTAGE

- Percentage $= \frac{\text{Part}}{\text{Whole}} \times 100$
- Increase % $= \frac{\text{Increase}}{\text{Original}} \times 100$
- Decrease % $= \frac{\text{Decrease}}{\text{Original}} \times 100$
- If a is increased by $p\%$,
  New value $= a\left(1 + \frac{p}{100}\right)$
- If a is decreased by $p\%$,
  New value $= a\left(1 - \frac{p}{100}\right)$

Example:

A number is increased by 20% and then decreased by 20%. Find net % change.

Solution:

Let number $= 100$

After increase $= 100 \times \frac{120}{100} = 120$

After decrease $= 120 \times \frac{80}{100} = 96$

Net change $= 96 - 100 = -4$

Net % change $= \frac{-4}{100} \times 100 = -4\%$

18. PROFIT, LOSS & DISCOUNT

- Profit = SP − CP
- Loss = CP − SP
- Profit % $= \frac{\text{Profit}}{\text{CP}} \times 100$
- Loss % $= \frac{\text{Loss}}{\text{CP}} \times 100$
- Discount = MP − SP
- Discount % $= \frac{\text{Discount}}{\text{MP}} \times 100$

Example:

CP of an article = ₹ 800

SP = ₹ 960

Find profit %.

Solution:

Profit = 960 − 800 = ₹ 160

Profit % $= \frac{160}{800} \times 100 = 20\%$

19. SIMPLE INTEREST

- Simple Interest (SI) $= \frac{P \times R \times T}{100}$
  (P = Principal, R = Rate%, T = Time in years)
- Amount (A) = P + SI
- SI for T years $= \frac{P \times R \times T}{100}$

Example:

Find SI on ₹ 5000 for 2 years at 6% p.a.

Solution:

SI $= \frac{5000 \times 6 \times 2}{100}$ = ₹ 600

Amount = 5000 + 600 = ₹ 5600

☆ Good Practice, Bright Future! ☆

20. COMPOUND INTEREST

- Amount after n years:

$$A = P\left(1 + \frac{R}{100}\right)^n$$

- Compound Interest (CI) = A − P
- For n years, $CI = P\left[\left(1 + \frac{R}{100}\right)^n - 1\right]$

Example:

Find amount and CI on ₹ 4000 for 2 years at 5% p.a. (compounded annually)

Solution:

$$A = 4000\left(1 + \frac{5}{100}\right)^2 = 4000\left(\frac{21}{20}\right)^2$$

$$= 4000 \times \frac{441}{400} = ₹\ 4410$$

$$CI = 4410 - 4000 = ₹\ 410$$

21. TIME, SPEED & DISTANCE

- Speed = $\frac{\text{Distance}}{\text{Time}}$ $\left(S = \frac{D}{T}\right)$
- Distance = Speed × Time $(D = S \times T)$
- Time = $\frac{\text{Distance}}{\text{Speed}}$ $\left(T = \frac{D}{S}\right)$
- If a covers distance D_1 with speed S_1 and next distance D_2 with speed S_2, then Average speed = $\dfrac{D_1 + D_2}{\frac{D_1}{S_1} + \frac{D_2}{S_2}}$

Example:

A car covers 120 km in 2 hours and next 180 km in 3 hours. Find average speed.

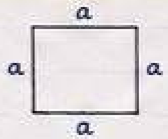
Solution:

$$\text{Avg. speed} = \frac{120 + 180}{\frac{120}{2} + \frac{180}{3}} = \frac{300}{60 + 60}$$

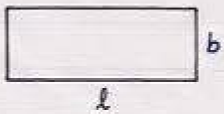
$$= \frac{300}{120} = 2.5 \text{ km/hr}$$

22. PERIMETER & AREA FORMULAS

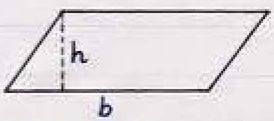
- Square
  Perimeter = 4a
  Area = a^2



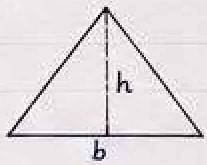
- Rectangle
  Perimeter = $2(\ell + b)$
  Area = $\ell \times b$



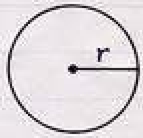
- Parallelogram
  Area = $b \times h$



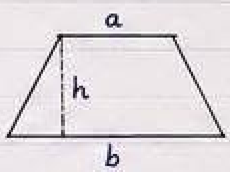
- Triangle
  Area = $\frac{1}{2}bh$



- Circle
  Circumfernce = $2\pi r$
  Area = πr^2



- Trapezium
  Area = $\frac{1}{2}(a + b)h$



23. FACTORS & MULTIPLES

- Factor of a number divides it exactly.
- LCM × HCF = Product of two numbers
- If $a = bq + r$, then
  $HCF(a, b) = HCF(b, r)$
  (Euclid's Division Lemma)
- A number is divisible by 2, 3, 5, 9, 10 if last digit rule or digit sum rule satisfies.

Divisibility Rules

2	→ Last digit is even (0, 2, 4, 6, 8)
3	→ Sum of digits is divisible by 3
5	→ Last digit is 0 or 5
9	→ Sum of digits is divisible by 9
10	→ Last digit is 0

Learn Formulas Daily, Solve Problems Easily! ☺

MATHEMATICS FORMULA SHEET

24. LOGARITHMS

- If $a^x = N$, then $\log_a N = x$
- $\log_a (MN) = \log_a M + \log_a N$
- $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$
- $\log_a (M^k) = k \log_a M$
- Change of base formula:

  $\log_a N = \frac{\log_b N}{\log_b a}$ $\quad (a>0, a\neq 1, b>0, b\neq 1)$

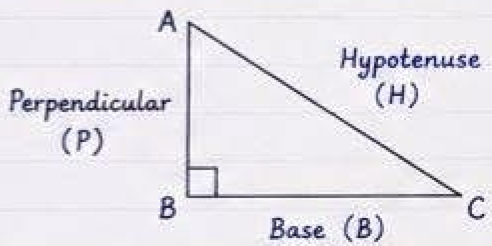
Example:

If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find $\log 6$.

Solution:

$$\begin{aligned} \log 6 &= \log(2\times 3) \\ &= \log 2 + \log 3 \\ &= 0.3010 + 0.4771 \\ &= 0.7781 \end{aligned}$$

25. TRIGONOMETRIC RATIOS



- $\sin A = \frac{P}{H}$
- $\cos A = \frac{B}{H}$
- $\tan A = \frac{P}{B}$
- $\text{cosec } A = \frac{H}{P}$
- $\sec A = \frac{H}{B}$
- $\cot A = \frac{B}{P}$

26. TRIGONOMETRIC IDENTITIES

- $\sin^2 A + \cos^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1 + \cot^2 A = \text{cosec}^2 A$
- $\tan A = \frac{\sin A}{\cos A}$
- $\cot A = \frac{\cos A}{\sin A}$
- $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- $\sin(A-B) = \sin A \cos B - \cos A \sin B$
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- $\cos(A-B) = \cos A \cos B + \sin A \sin B$
- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

27. QUADRATIC FORMULAS

- Quadratic Equation: $ax^2 + bx + c = 0 \quad (a \neq 0)$
- Roots:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

- Sum of roots $= \alpha + \beta = -\frac{b}{a}$
- Product of roots $= \alpha\beta = \frac{c}{a}$

Example:

Solve $2x^2 - 5x + 3 = 0$

Solution:

$a = 2,\ b = -5,\ c = 3$

$$x = \frac{5 \pm \sqrt{25 - 24}}{4} = \frac{5 \pm 1}{4}$$

$$x = \frac{6}{4}, \frac{4}{4} = \frac{3}{2}, 1$$

28. ARITHMETIC PROGRESSION (A.P.)

- n^{th} term: $a_n = a + (n-1)d$
- Sum of first n terms: $S_n = \frac{n}{2}[2a + (n-1)d]$
- Common difference: $d = a_2 - a_1$

Example:

If $a = 3$, $d = 4$, find S_{10}.

Solution:

$$\begin{aligned} S_{10} &= \frac{10}{2}[2(3) + (10-1)4] \\ &= 5[6 + 36] = 5 \times 42 = 210 \end{aligned}$$

☆ *Practice Regularly, Improve Continuously!* ☆

29. GEOMETRIC PROGRESSION (G.P.)

- n^{th} term: $a_n = ar^{n-1}$
- Sum of first n terms $(r \neq 1)$:

$$S_n = a\,\frac{r^n - 1}{r - 1}$$

- Sum to infinity $(|r| < 1)$: $S_\infty = \frac{a}{1-r}$
- Common ratio: $r = \frac{a_2}{a_1}$

Example:

If $a = 3$, $r = 2$, find S_5.

Solution:

$$S_5 = 3\,\frac{2^5 - 1}{2 - 1} = 3(32 - 1)$$

$$= 3 \times 31 = 93$$

30. COORDINATE GEOMETRY FORMULAS

- Distance between (x_1, y_1) and (x_2, y_2):

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

- Midpoint of (x_1, y_1) and (x_2, y_2):

$$M\left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right)$$

- Section Formula (internal division):

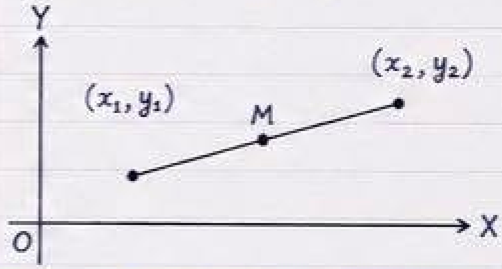
$$P\left(\frac{mx_2 + nx_1}{m + n},\ \frac{my_2 + ny_1}{m + n}\right)$$

Example:

Find midpoint of $A(2, -3)$ and $B(8, 5)$.

Solution:

$$M\left(\frac{2 + 8}{2},\ \frac{-3 + 5}{2}\right) = (5, 1)$$



31. AREA FORMULAS

- Square $= a^2$
- Rectangle $= l \times b$
- Parallelogram $= b \times h$
- Triangle $= \frac{1}{2}\,bh$
- Trapezium $= \frac{1}{2}(a + b)h$
- Circle $= \pi r^2$
- Sector of circle $= \frac{\theta}{360^\circ}\pi r^2$
  (θ in degrees)

32. VOLUME FORMULAS

- Cube $= a^3$
- Cuboid $= l \times b \times h$
- Cylinder $= \pi r^2 h$
- Cone $= \frac{1}{3}\pi r^2 h$
- Sphere $= \frac{4}{3}\pi r^3$
- Hemisphere $= \frac{2}{3}\pi r^3$

33. MISCELLANEOUS

- $|a| = \sqrt{a^2}$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- If $a^m = a^n$, then $m = n$
  $(a > 0,\ a \neq 1)$

34. IMPORTANT CONSTANTS

- $\pi = 3.1416$
- $\sqrt{2} = 1.4142$
- $\sqrt{3} = 1.7321$
- g (acc. due to gravity) $= 9.8\ m/s^2$
- $1^\circ = \frac{\pi}{180}$ rad
- 1 rad $= \frac{180^\circ}{\pi}$

Tip:
Memorize formulas,
Understand concepts,
Practice questions daily.

Strong Concepts + Smart Practice = Top Results! ☺

MATHEMATICS FORMULA SHEET

35. SUM & PRODUCT FORMULAS

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a+b)(a-b) = a^2 - b^2$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Example :

If $a = 2$, $b = 3$, find $(a+b)^2$.

Solution :

$$\begin{aligned}(a+b)^2 &= a^2 + 2ab + b^2 \\ &= 2^2 + 2(2)(3) + 3^2 \\ &= 4 + 12 + 9 = 25\end{aligned}$$

36. INDICES (EXPONENTS) LAWS

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n} \quad (a \neq 0)$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0)$
- $a^0 = 1 \quad (a \neq 0)$
- $a^{-n} = \frac{1}{a^n}$

Example :

Simplify : $\dfrac{(3^2 \times 3^{-5})}{3^{-1}}$

Solution :

$$\begin{aligned}&= \frac{3^{2-5}}{3^{-1}} \\ &= 3^{-3} \div 3^{-1} \\ &= 3^{-3+1} = 3^{-2} = \frac{1}{9}\end{aligned}$$

37. SURDS (RATIONALISATION)

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab} \quad (a, b \geq 0)$
- $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}} \quad (b > 0)$
- Conjugate of $(a + \sqrt{b})$ is $(a - \sqrt{b})$
- $\dfrac{1}{a + \sqrt{b}} = \dfrac{a - \sqrt{b}}{a^2 - b} \quad (a^2 \neq b)$

Example :

Rationalise : $\dfrac{1}{2 + \sqrt{3}}$

Solution :

$$\frac{1}{2+\sqrt{3}} \times \frac{(2-\sqrt{3})}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$$

38. LOGARITHM LAWS

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a (MN) = \log_a M + \log_a N$
- $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$
- $\log_a (M^k) = k \log_a M$
- Change of base formula :

  $\log_a N = \dfrac{\log_b N}{\log_b a} \quad (a>0, a \neq 1, b>0, b \neq 1)$

Example :

If $\log_2 5 = 2.3219$ and $\log_2 3 = 1.5850$, find $\log_2 15$.

Solution :

$$\begin{aligned}\log_2 15 &= \log_2 (3 \times 5) \\ &= \log_2 3 + \log_2 5 \\ &= 1.5850 + 2.3219 \\ &= 3.9069\end{aligned}$$

☆ Math is Fun, Formula Makes it Easy! ☆

39. POLYNOMIALS

- Degree of a polynomial: highest power of the variable.
- Zero of $p(x)$: If $p(a) = 0$, then $(x - a)$ is a factor.
- Remainder Theorem: If a polynomial $p(x)$ is divided by $(x - a)$, remainder $= p(a)$.
- Factor Theorem: $(x - a)$ is a factor of $p(x)$ iff $p(a) = 0$.
- Sum of zeros $= -\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2}$ (for $ax^2 + bx + c$).
- Product of zeros $= \dfrac{\text{constant term}}{\text{coefficient of } x^2}$ (for $ax^2 + bx + c$)

Example:

If $p(x) = x^2 - 5x + 6$, find the sum and product of its zeros.

Solution:

For $ax^2 + bx + c$:

$$\text{Sum} = -\frac{b}{a} = \frac{-(-5)}{1} = 5$$

$$\text{Product} = \frac{c}{a} = \frac{6}{1} = 6$$

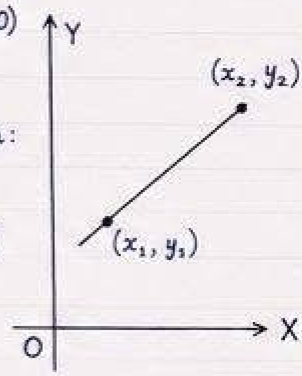
40. LINEAR EQUATIONS (TWO VARIABLES)

- Standard form: $ax + by + c = 0$ (a, b not both 0)
- Slope of line: $m = -\dfrac{a}{b}$ $(b \neq 0)$
- Equation of line through (x_1, y_1) with slope m:

$$y - y_1 = m(x - x_1)$$

- Equation of line through two points (x_1, y_1) and (x_2, y_2):

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



Example:

Find equation of line through $(2, 3)$ and $(6, 7)$.

Solution:

$$y - 3 = \frac{7 - 3}{6 - 2}(x - 2)$$

$$y - 3 = 1(x - 2)$$

$$y - 3 = x - 2$$

$$\boxed{y = x + 1}$$

41. SYSTEM OF LINEAR EQUATIONS

- For equations $a_1x + b_1y + c_1 = 0$
  $a_2x + b_2y + c_2 = 0$
- Unique solution if $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$
- No solution if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$
- Infinitely many solutions if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

42. AP (ARITHMETIC PROGRESSION)

- n^{th} term: $a_n = a + (n-1)d$
- Sum of first n terms:
  $S_n = \dfrac{n}{2}[2a + (n-1)d]$
- If last term $= \ell$, then
  $\ell = a + (n-1)d$
- Number of terms:
  $n = \dfrac{\ell - a}{d} + 1$

43. GP (GEOMETRIC PROGRESSION)

- n^{th} term: $a_n = ar^{n-1}$
- Sum of first n terms:
  $S_n = a\dfrac{(r^n - 1)}{r - 1}$ $(r \neq 1)$
- Sum to infinity $(|r| < 1)$:
  $S_\infty = \dfrac{a}{1 - r}$
- Common ratio: $r = \dfrac{a_2}{a_1}$

44. SOME USEFUL INEQUALITIES

- $(a + b)^2 \geq 0 \Rightarrow a^2 + b^2 \geq 2ab$
- $(a - b)^2 \geq 0 \Rightarrow a^2 + b^2 \geq 2ab$
- For $a, b > 0$:
  $a + b \geq 2\sqrt{ab}$ (AM ≥ GM)
  $a^2 + b^2 \geq 2ab$

45. BASIC NUMBER THEORY

- $HCF(a, b) \times LCM(a, b) = a \times b$
- A number is prime if it has exactly two factors: 1 and itself.
- 1 is neither prime nor composite.
- Every composite number can be written as a product of primes (Fundamental Theorem of Arithmetie).

Tip:

Learn formulas, Practice problems, Revise regularly and stay confident! ☺

Practice + Patience + Positive Attitude = Math Success! ☺

MATHEMATICS FORMULA SHEET

46. DATA HANDLING FORMULAS

- Mean (Average) : $\bar{x} = \frac{\Sigma x_i}{n}$
- Mean of grouped data : $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$
  (f_i = frequency, x_i = class mark)
- Median (Ungrouped) : Middle value when data is arranged in order.
- Mode : Value that occurs most frequently.
- Range = Maximum value − Minimum value
- Standard Deviation (σ) : $\sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$
- Variance (σ^2) : $\sigma^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$

Example :

Find mean of 2, 4, 6, 8, 10.

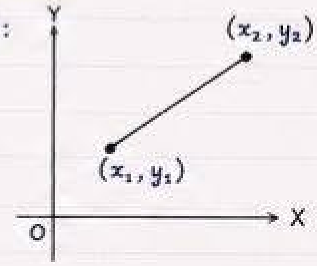
Solution :

$$\bar{x} = \frac{2 + 4 + 6 + 8 + 10}{5}$$

$$= \frac{30}{5} = 6$$

47. CO-ORDINATE GEOMETRY – DISTANCE FORMULAS

- Distance between two points (x_1, y_1) and (x_2, y_2) :
  $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
- Distance of a point (x, y) from origin $(0, 0)$:
  $$d = \sqrt{x^2 + y^2}$$



Example :

Find distance between A(1, 2) and B(4, 6).

Solution :

$$d = \sqrt{(4-1)^2 + (6-2)^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16} = \sqrt{25} = 5$$

48. SECTION FORMULA

- If a point $P(x, y)$ divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ (i.e., $AP : PB = m : n$), then

$$\boxed{P\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)}$$

Example :

Find the point which divides the line segment joining A(2, 3) and B(6, 9) in the ratio 1 : 2.

Solution :

$$P\left(\frac{1 \times 6 + 2 \times 2}{1 + 2}, \frac{1 \times 9 + 2 \times 3}{1 + 2}\right)$$

$$= \left(\frac{10}{3}, \frac{15}{3}\right) = \left(\frac{10}{3}, 5\right)$$

49. MIDPOINT FORMULA

- Midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$\boxed{M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

Example :

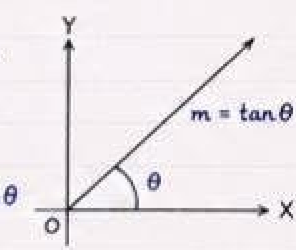
Find midpoint of A(3, 7) and B(9, −1).

Solution :

$$M\left(\frac{3 + 9}{2}, \frac{7 + (-1)}{2}\right) = (6, 3)$$

50. SLOPE OF A LINE

- Slope (m) of line passing through (x_1, y_1) and (x_2, y_2) :
  $$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (x_2 \neq x_1)$$
- If line makes an angle θ with +ve X-axis, then $m = \tan\theta$



Example :

Find slope of line through (2, 3) and (5, 11).

Solution :

$$m = \frac{11 - 3}{5 - 2} = \frac{8}{3}$$

☆ Practice Hard, Stay Consistent, Achieve Success! ☆

51. CONCURRENT LINES

- Two lines are concurrent if they intersect at a point.
- Condition for concurrency of lines:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

(for equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$)

Example:

Check if the lines $2x + 3y + 4 = 0$ and $4x + 6y + 5 = 0$ are concurrent.

Solution:

$$\frac{2}{4} = \frac{3}{6} = \frac{1}{2} \neq \frac{4}{5}$$

Hence, lines are concurrent.

52. PARALLEL LINES

- Two lines are parallel if they never intersect.
- Condition for parallel lines:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(for equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$)

Example:

Check if the lines $2x + 3y + 4 = 0$ and $4x + 6y + 8 = 0$ are parallel.

Solution:

$$\frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{1}{2}$$

Hence, lines are parallel.

53. PERPENDICULAR LINES

- Two lines are perpendicular if they intersect at right angle (90°).
- Condition for perpendicular lines:

$$a_1a_2 + b_1b_2 = 0$$

(for equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$)

Example:

Check if $2x + 3y + 1 = 0$ and $3x - 2y + 5 = 0$ are perpendicular.

Solution:

$$2 \times 3 + 3 \times (-2) = 6 - 6 = 0$$

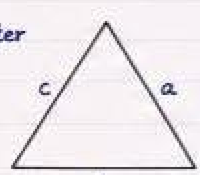
Hence, lines are perpendicular.

54. HERON'S FORMULA

- For a triangle with sides a, b, c and semi-perimeter

$$s = \frac{a+b+c}{2}$$

$$\text{Area } (\Delta) = \sqrt{s(s-a)(s-b)(s-c)}$$



Example:

Find area of triangle with sides 13 cm, 14 cm and 15 cm.

Solution:

$$s = \frac{13 + 14 + 15}{2} = 21$$

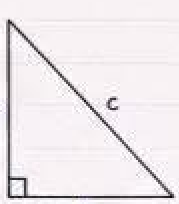
$$\text{Area} = \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84 \text{ cm}^2$$

55. PYTHAGOREAN THEOREM

- In a right-angled triangle, $(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

i.e., $c^2 = a^2 + b^2$



Example:

In right triangle, if $a = 5$ cm and $b = 12$ cm, find c.

Solution:

$$c = \sqrt{a^2 + b^2} = \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144} = \sqrt{169} = 13 \text{ cm}$$

56. SOME IMPORTANT IDENTITIES

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a-b)(a+b)$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Tip:

Learn formulas,
Practice problems,
Revise regularly
and be confident! ☺

Smart Work + Regular Practice = Excellent Results! ☺

MATHEMATICS FORMULA SHEET

57. FACTORISATION FORMULAS

- $x^2 - y^2 = (x - y)(x + y)$
- $x^2 + 2xy + y^2 = (x + y)^2$
- $x^2 - 2xy + y^2 = (x - y)^2$
- $x^2 + y^2 + 2xy + z^2 + 2yz + 2zx = (x + y + z)^2$
- $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
- $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
- $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Example :

Factorise : $x^2 - 9$

Solution :

$x^2 - 9 = x^2 - 3^2$

$= (x - 3)(x + 3)$

58. LINEAR INEQUALITIES

- If $ax + b > 0$ and $a > 0$, then $x > -\frac{b}{a}$
- If $ax + b > 0$ and $a < 0$, then $x < -\frac{b}{a}$
- If $ax + b < 0$ and $a > 0$, then $x < -\frac{b}{a}$
- If $ax + b < 0$ and $a < 0$, then $x > -\frac{b}{a}$

Example :

Solve $3x - 6 > 0$.

Solution :

$3x > 6$

$x > 2$

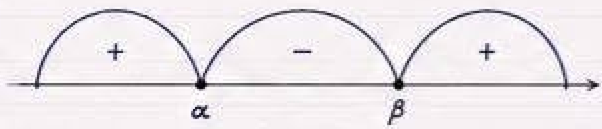
59. QUADRATIC INEQUALITY

- If $ax^2 + bx + c > 0$

  $(a > 0) \Rightarrow x < \alpha$ or $x > \beta$
- If $ax^2 + bx + c < 0$

  $(a > 0) \Rightarrow \alpha < x < \beta$

where α and β are the roots of $ax^2 + bx + c = 0$ and $\alpha < \beta$.



$(a > 0)$

60. STATEMENT FORMULAS (LOGIC)

- Negation : $\sim p \rightarrow$ 'not p'
- Conjunction : $p \wedge q \rightarrow$ 'p and q'
- Disjunction : $p \vee q \rightarrow$ 'p or q'
- Conditional : $p \Rightarrow q \rightarrow$ 'if p then q'
- Biconditional : $p \Leftrightarrow q \rightarrow$ 'p if and only if q'

p	q	$p \wedge q$	$p \vee q$	$p \Rightarrow q$	$p \Leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

61. SOLUTION OF LINEAR EQUATIONS

- For $ax + b = 0$

  $(a \neq 0)$

  $\boxed{x = -\frac{b}{a}}$

Example :

Solve $5x + 15 = 0$.

Solution :

$5x = -15$

$x = -3$

62. SUM OF n NATURAL NUMBERS

- $1 + 2 + 3 + \cdots + n = \frac{n(n + 1)}{2}$

Example :

Find sum of first 20 natural numbers.

Solution : $S = \frac{20(20 + 1)}{2} = \frac{20 \times 21}{2} = 210$

☆ Regular Practice + Strong Concepts = Math Excellence! ☆

63. SUM OF SQUARES

- $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$
- $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \dfrac{n(2n-1)(2n+1)}{3}$
- $2^2 + 4^2 + 6^2 + \cdots + (2n)^2 = \dfrac{n(n+1)(2n+1)}{3}$

Example :

Find $1^2 + 2^2 + \cdots + 10^2$.

Solution :

$$= \frac{10(10+1)(2 \times 10 + 1)}{6}$$
$$= \frac{10 \times 11 \times 21}{6} = 385$$

64. SUM OF CUBES

- $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$
- $1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3 = n^2(2n^2 - 1)$
- $2^3 + 4^3 + 6^3 + \cdots + (2n)^3 = n^2(2n^2 + 1)$

Example :

Find $1^3 + 2^3 + \cdots + 8^3$.

Solution :

$$= \left[\frac{8(8+1)}{2}\right]^2 = 36^2 = 1296$$

65. AP (ARITHMETIC PROGRESSION)

- n^{th} term: $a_n = a + (n-1)d$
- Sum of first n terms: $S_n = \dfrac{n}{2}[2a + (n-1)d]$
- If t_1, t_2, t_3 are in A.P., then $2t_2 = t_1 + t_3$

Example :

In an A.P., $a = 3$, $d = 4$. Find S_{15}.

Solution :

$$S_{15} = \frac{15}{2}[2(3) + (15-1)4]$$
$$= \frac{15}{2}[6 + 56] = \frac{15}{2} \times 62 = 465$$

66. GP (GEOMETRIC PROGRESSION)

- n^{th} term: $a_n = ar^{n-1}$
- Sum of first n terms $(r \neq 1)$: $S_n = a\dfrac{(r^n - 1)}{(r-1)}$
- Sum to infinity $(|r| < 1)$: $S_\infty = \dfrac{a}{1-r}$
- Product of first n terms: $P_n = a^n r^{\frac{n(n-1)}{2}}$

Example :

In a G.P., $a = 2$, $r = 3$. Find S_5.

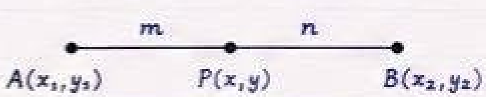
Solution :

$$S_5 = 2\,\frac{(3^5 - 1)}{3 - 1} = 2\,\frac{(243-1)}{2} = 242$$

67. COORDINATE GEOMETRY – SECTION FORMULA

- If $P(x, y)$ divides line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in ratio $m : n$, then

$$\boxed{P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)}$$



69. AREA BETWEEN TWO CURVES

If $y = f(x)$ is above $y = g(x)$ on $[a, b]$,

then area between curves

$$= \int_a^b [f(x) - g(x)]\,dx$$

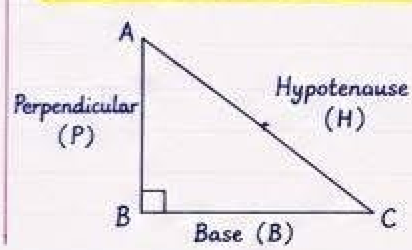
99. INTEGATIVES (BASIC)

- $\dfrac{d}{dx}(x^n) = nx^{n-1}$
- $\dfrac{d}{dx}(\sin x) = \cos x$
- $\dfrac{d}{dx}(\cos x) = -\sin x$
- $\dfrac{d}{dx}(e^x) = e^x$
- $\dfrac{d}{dx}(\log x) = \dfrac{1}{x}$
- $\dfrac{d}{dx}(\tan x) = \sec^2 x$

70. INTEGRALS (BASIC)

- $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
- $\int \dfrac{1}{x}\,dx = \log|x| + C$
- $\int e^x\,dx = e^x + C$
- $\int \sin x\,dx = -\cos x + C$
- $\int \cos x\,dx = \sin x + C$
- $\int \sec^2 x\,dx = \tan x + C$

71. TRIGONOMETRIC RATIOS



$\sin A = \dfrac{P}{H}$

$\cos A = \dfrac{B}{H}$

$\tan A = \dfrac{P}{B}$

$\cot A = \dfrac{B}{P}$

$\sec A = \dfrac{H}{B}$

$\text{cosec } A = \dfrac{H}{P}$

Revise Regularly, Solve Smartly, Score Highly! ☺